Discontinuity Propagation in Hybrid System Simulation

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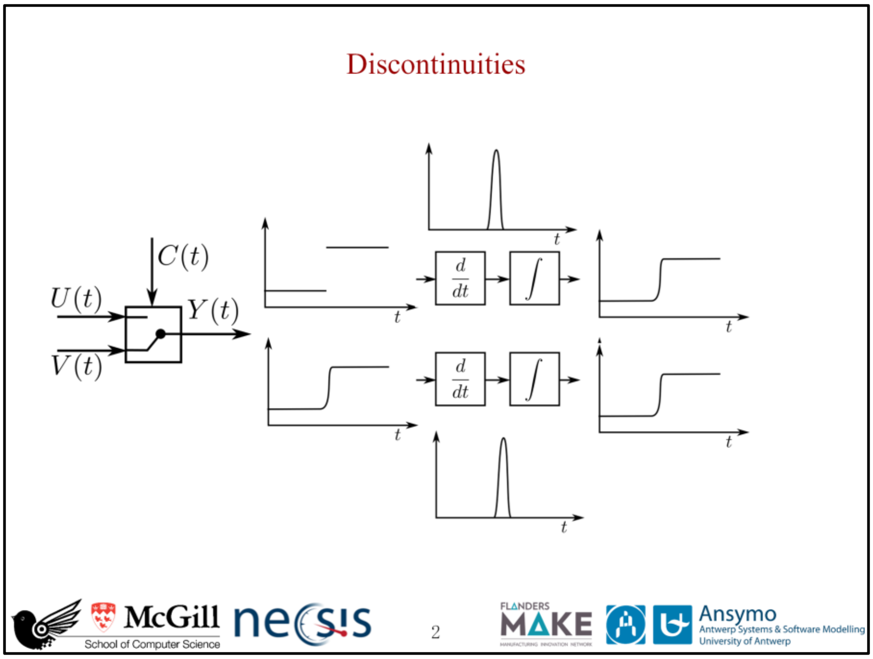
Abstract

Physical systems are inherently continuous.
As such, creating detailed models of these is difficult, if not unfeasible.
Fortunately, hybrid models that abstract away fast behaviors can greatly simplify the study of such systems.
For example, impulses are often used as an intuitive abstraction of a very stiff change in the state of the system that occurs, for instance, in collisions.
Given the mathematical peculiarities of impulses, it becomes difficult to define properly what kind of numerical operations can be performed on impulses, for the purpose of simulation of differential equations.
In this work, we extend the Causal Block Diagrams (CBD) formalism with impulses and we give a theoretical foundation for each operation in terms of distribution theory.
This enables the invariants such as the fundamental theorem of calculus to be kept even in the presence of impulses and discontinuities.
The result is a formalism that can accurately simulation a class of hybrid systems.
We show a simple example of a bouncing ball and study the performance and accuracy of the simulator when impulses are used.



Motivation:

We start with a discontinuity.
In some system, this signal represents the product of an abstraction process.
For example, in the circuit domain, when a switch is turned on, there are fast oscillations in this signal but the modelers did not care to include those in their model, and so they used a pure discontinuity.
Similarly, if a ball is suspended by some wire, and that wire is cut, the resultant force acting in the ball will be best abstracted as a pure discontinuity.

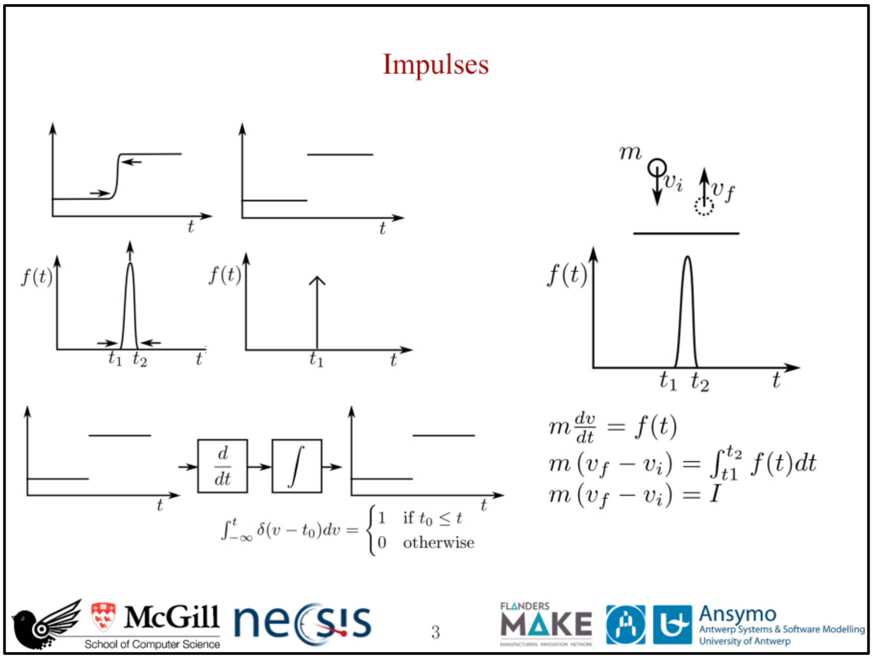
CBDs incorporate a very neat block that allows one to make discontinuities: the switch block.

CBDs also include all sorts of other algebraic and differential blocks.

Let us focus on a particular arrangement of those blocks: a derivative of a discontinuity, followed by an integral.
What would be the meaning of this if the discontinuity here is to be seen as a true discontinuity?

Include the plot of the non-abstracted discontinuity.

Observe that we lost the discontinuity, under the fundamental theorem of calculus.
You might argue that the fundamental theorem is applicable only to differentiable functions and that the discontinuity, at least in the pure sense, is not differentiable.



This is to show that these switches are not just of theoretical value but that these are used in everyday modeling.

So, a discontinuity is an abstraction of time where the fast transients of the system are ignored.
Let us look at an example where the very same kind of abstraction yields a different signal.

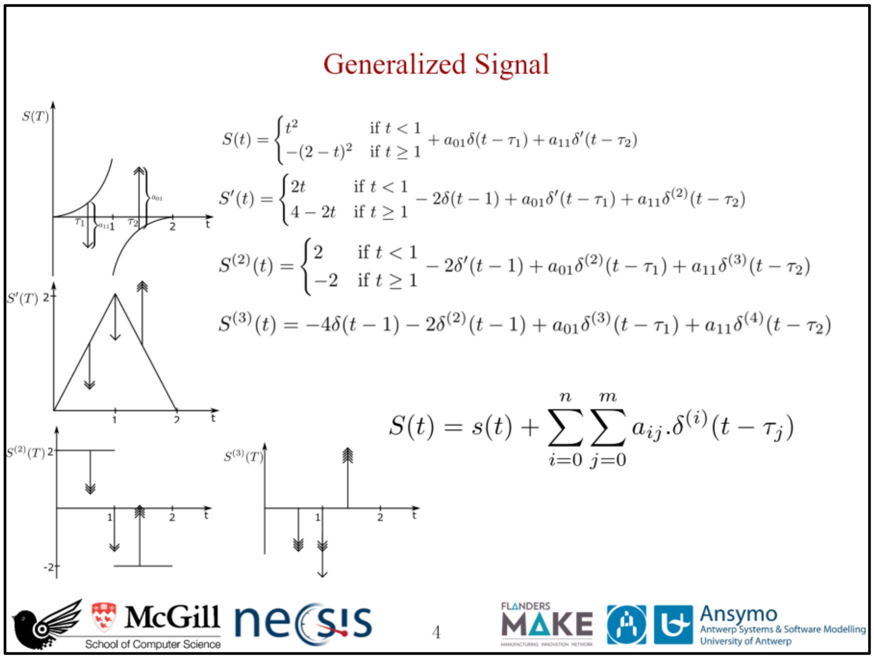
[Derive the bouncing ball model using the concept of impulsive force.]

Show that, by abstracting the interval of time in which the impulse occurs, what we get is a discontinuous change in the velocity of the bal, originated by an impulse.
Show that this is very intuitive for modeling these systems and that by incorporating this concept into the CBDs we can simulate more complex systems.

Describe what is an impulse.

The most straight forward way of getting an impulse is by differentiating a discontinuity (show an example with this).

This shows that we cannot properly abstract discrete changes in a system without addressing the derivative and integral operators of those signals as well.



Show an example of a generalized signal and its derivatives.
Explain each element in detail.

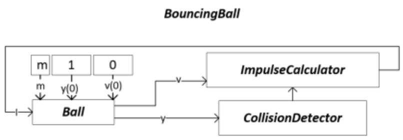
Remark the extra impulse arising at the derivative signal.

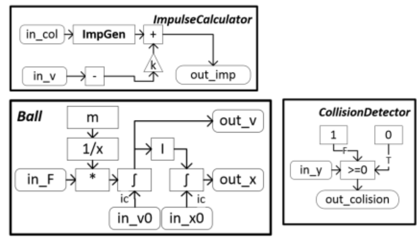
Explain how the information of the discontinuity is always present by looking at the derivatives of the signal.
This is regardless of whether we have the left limit or the right limit, i.e., regardless of the condition.

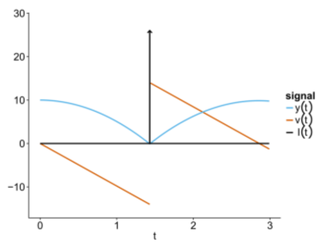
For this signal we end up with a train of impulses, but this is not necessarily the case.
If this were a sin function, when there would always be a derivative of any order.

Show what does the signal look like in the general case.

Bouncing Ball







Δt	MSE Hybrid CBDs	MSE Normal CBDs
10^{-2}	9.188382×10^{-3}	2.012778×10^{-2}
10^{-3}	1.476189×10^{-4}	2.93385×10^{-4}
10^{-4}	1.185938×10^{-6}	2.474646×10^{-6}

MSE	Exec Time Hybrid CBDs	Exec Time Normal CBDs
9.128132×10^{-3}	2.887s (238%)	1.212s (100%)

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Tell what has been done: the implementation of each of the blocks of the CBDs supporting impulses.
We do not lose information about the discontinuities and we get more accurate

Show the bouncing ball model and the simulation results.
Highlight the accuracy gains under fixed step size and the performance problems.

Discuss the future work.

Thank you!