MSDL
Modelling, Simulation and Design Lab

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Stable Optimization of Co-simulation: A Switched Systems Approach

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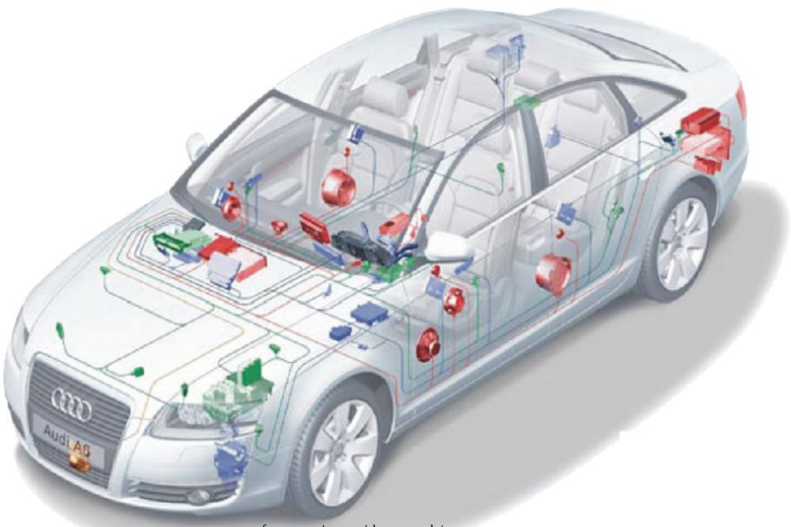
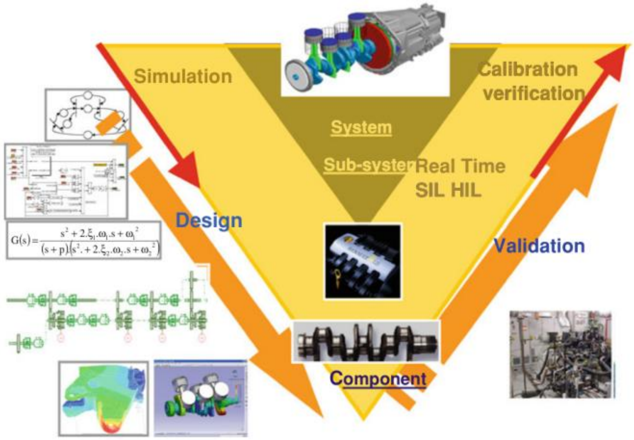
Why Co-simulation? – Applications Perspective

- Increasing Complexity
  - Interacting heterogeneous components
- Competitive Market
- Concurrent Development
  - Late Integration Problems
  - Contracts
- Independently Developed Sub-systems
  - Specialized Teams
  - External Suppliers
- "Holy Grail":
  - Integration at **every** stage of development

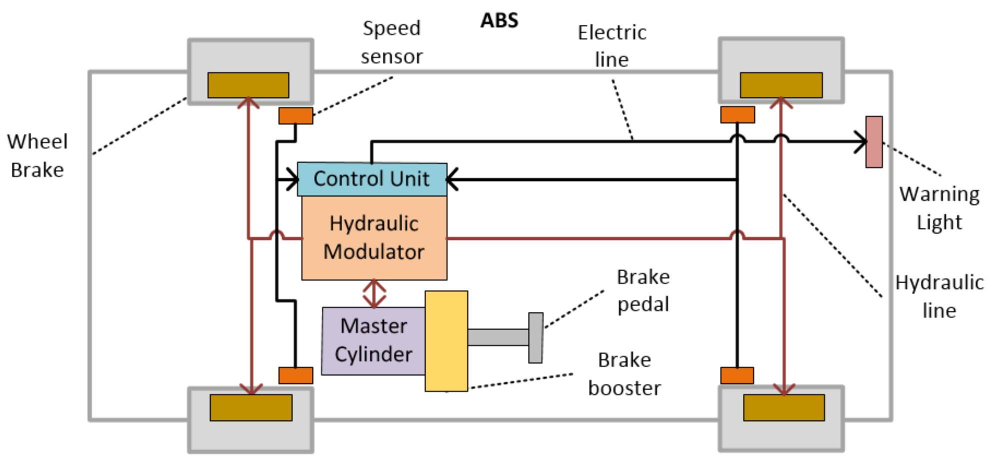


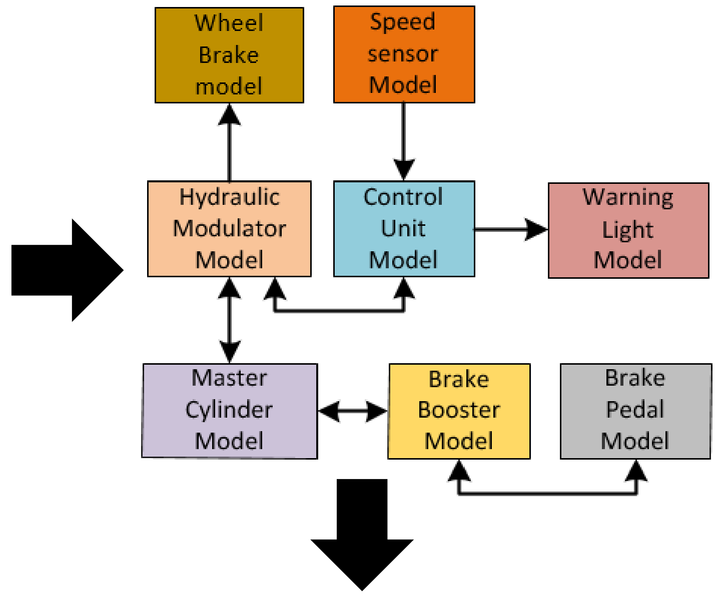
from www.imes.uni-hannover.de/

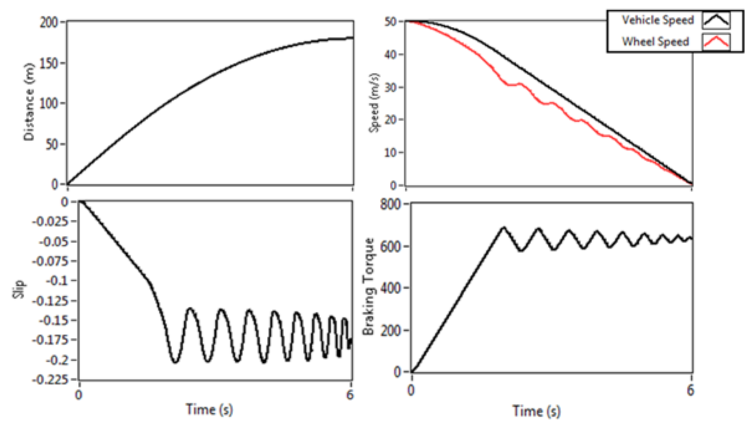


from http://doi.org/10.1007/s00366-012-0286-6

Modelling and Simulation





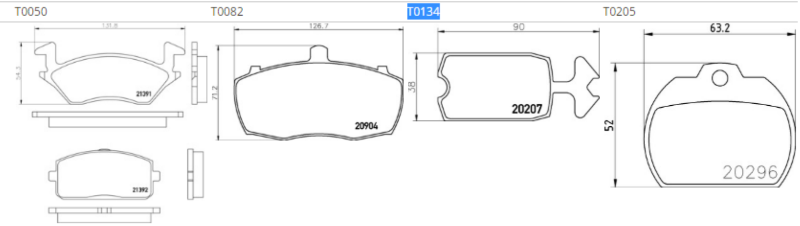


- Modelling and Simulation techniques help
  - Models of environment capture the assumptions
  - Data sheets and Contracts provide parameters and operating ranges.

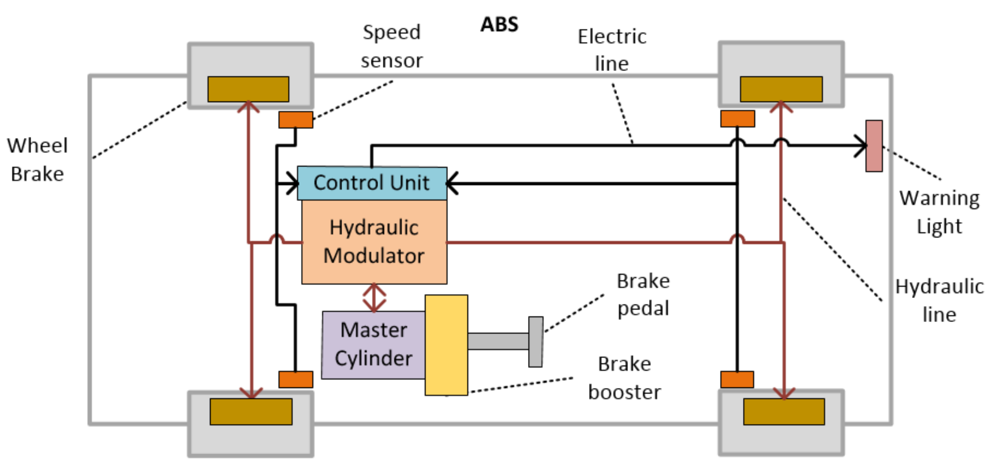
IP Protection

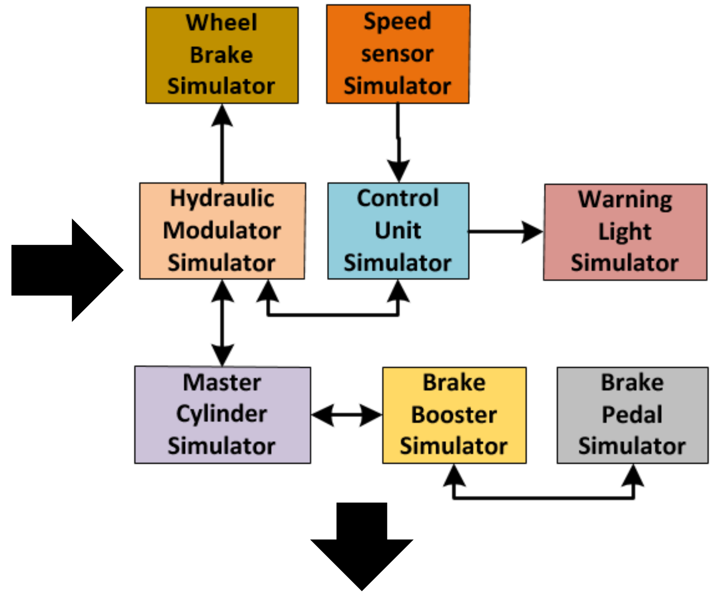
- Design Space Exploration
  - with supplied components and their parameters
- More information about components means more optimization
- Suppliers want to protect IP
  - No detailed models provided
  - Catalogs may lack necessary information
    - Applicability range or Validity Frame. Ex:
      - Temperature
      - Response tables from lab experiments
      - Wear &Tear dynamics

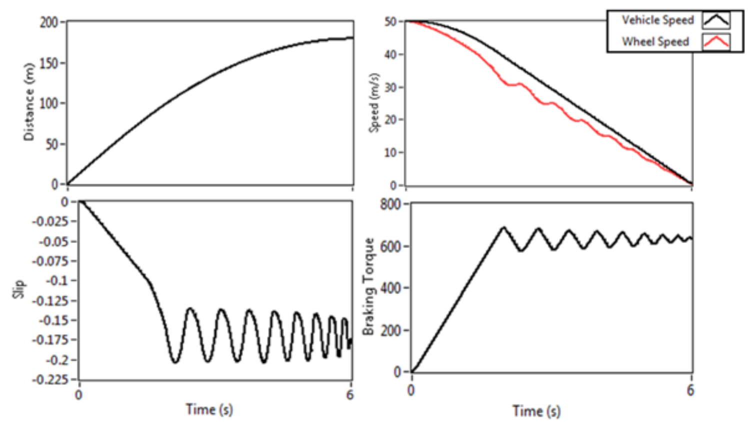
COMPARISON LIST

Product	Brake Pad Set, disc brake	Brake Pad Set, disc brake	Brake Pad Set, disc brake	Brake Pad Set, disc brake
Article number	T0050	T0082	T0134	T0205
Image				
Brake System	Akebono	Lockheed	Citroen	Lucas
Axle	Front Axle	Front Axle	Front Axle	Front Axle
Width [mm]	97.5 - 131.8	126.7	90	63.2
Height [mm]	44 - 54.3	71.2	38	52
usage number	• 21391 • 21392 • 7107D180	• 20904 • 796D61	20207	• 20296 • 7000D71 • 7022D88
Thickness [mm]	13.5	18	13	16.2
Wear Warning Contact	excl. wear warning contact	excl. wear warning contact	excl. wear warning contact	excl. wear warning contact
replacement part	No	No	No	No
Country Version	• Great Britain • Italy	• Great Britain • Italy	• Great Britain • Italy	• Great Britain • Italy

Co-simulation







- Simulators as mockups of supplied components
  - Provided by supplier as virtual black boxes (e.g., binary, or web service)
- Co-simulation: orchestration of coupled black box simulators, each standing for a sub-system, with the purpose of studying the salient characteristics of the coupled system.
  - Enable early DSE with supplied simulators.

Simulators

$$S_i = \langle X_i, U_i, Y_i, \delta_i, \lambda_i, x_i(0), \phi_{U_i} \rangle$$
$$\delta_i : \mathbb{R} \times \mathbb{R} \times X_i \times U_i \rightarrow X_i$$
$$\lambda_i : \mathbb{R} \times X_i \times U_i \rightarrow Y_i \text{ or } \mathbb{R} \times X_i \rightarrow Y_i$$
$$x_i(0) \in X_i$$
$$\phi_{U_i} : \mathbb{R} \times U_i \times \ldots \times U_i \rightarrow U_i$$

Simulator

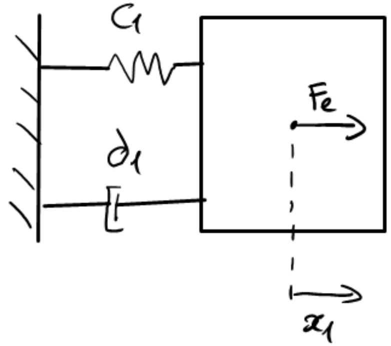
Model

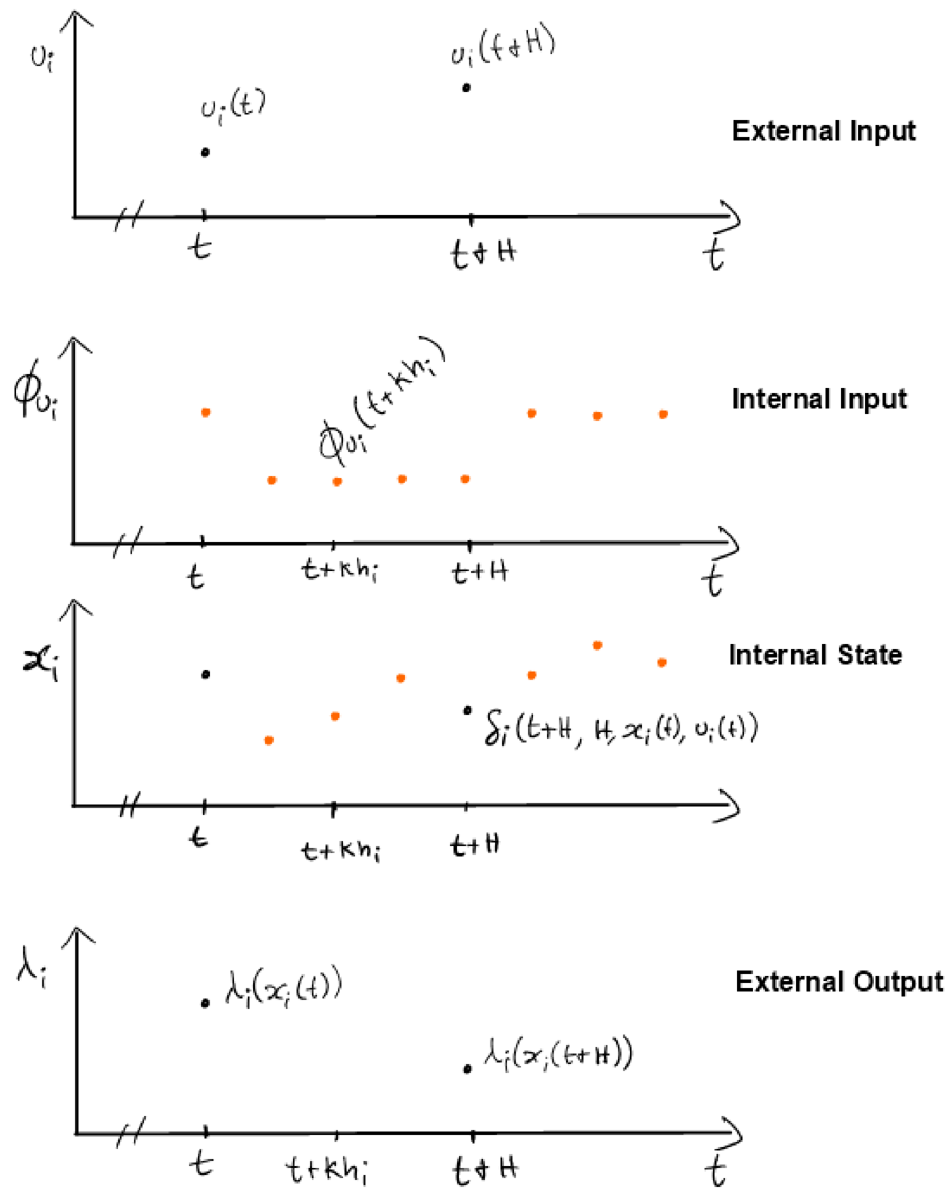
$$m_1 \cdot \ddot{x}_1 = -c_1 \cdot x_1 - d_1 \cdot \dot{x}_1 + F_e$$
$$x_1(0) = p_1; \quad \dot{x}_1(0) = s_1$$

+

Solver

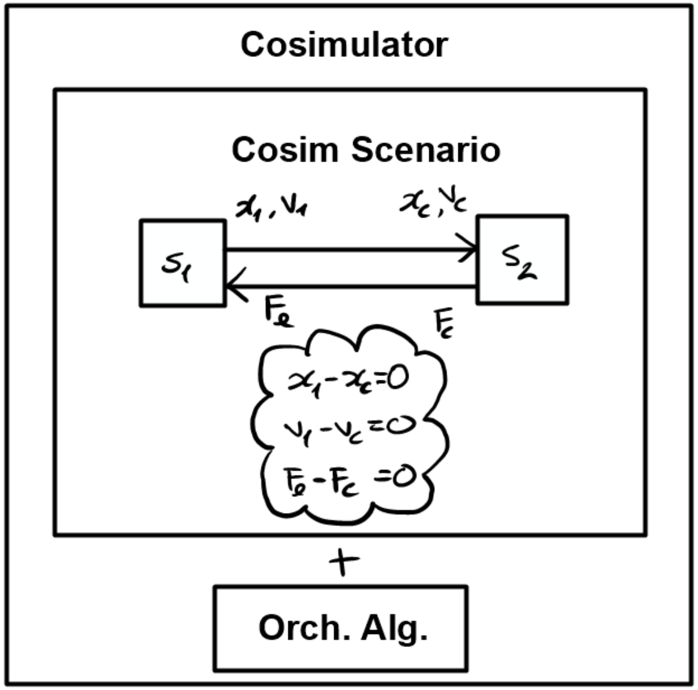
$$\tilde{x}(t+h) := \tilde{x}(t) + f(\tilde{x}(t), u(t)) \cdot h$$
$$\tilde{x}(0) := x(0)$$

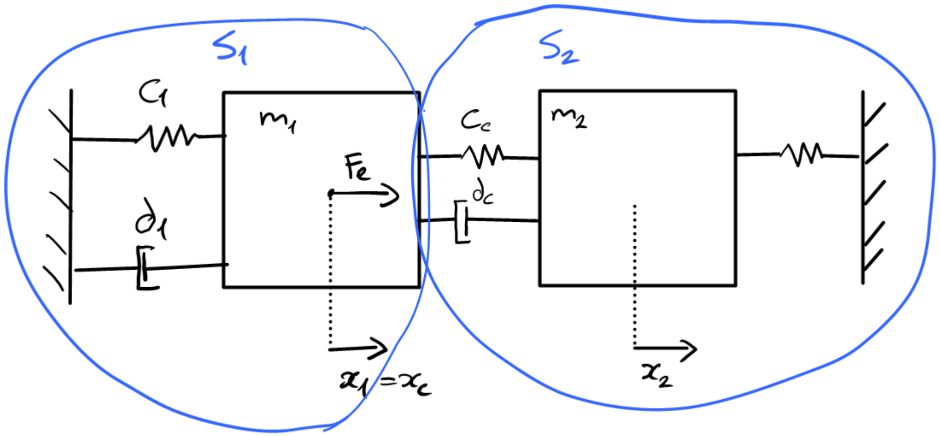


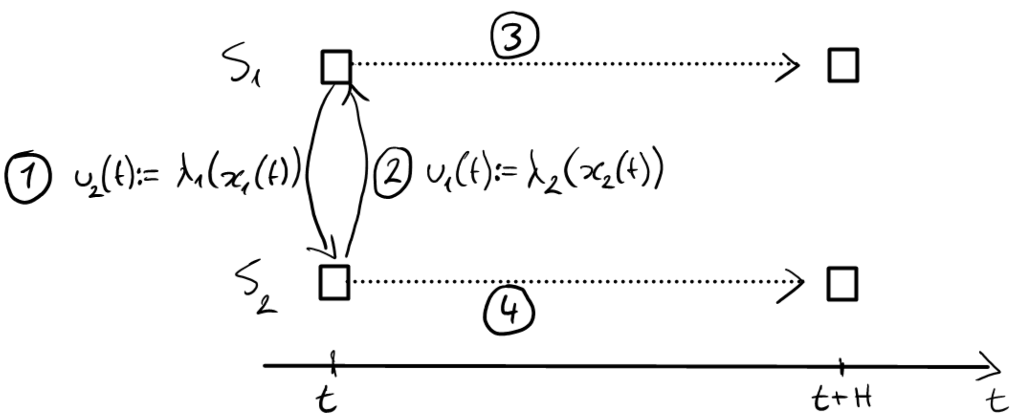


Co-simulation

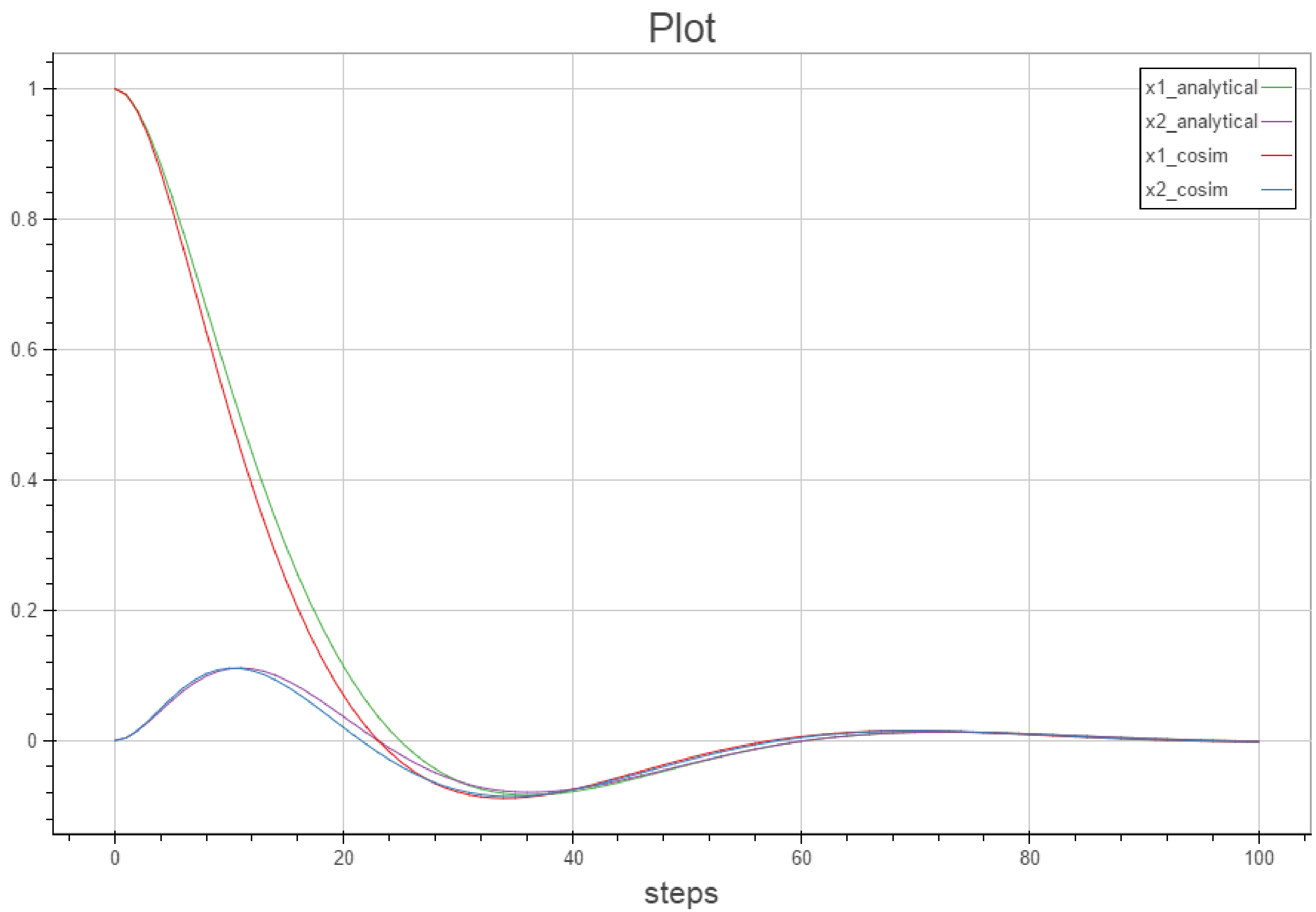
$$CS = \langle S, L \rangle$$
$$S = (S_1, \dots S_n)$$
$$L : Y_1 \times \dots \times Y_n \times U_1 \times \dots \times U_n \to \mathbb{R}^n$$
$$L(y_1, \dots, y_n, u_1, \dots, u_n) = \bar{0}$$







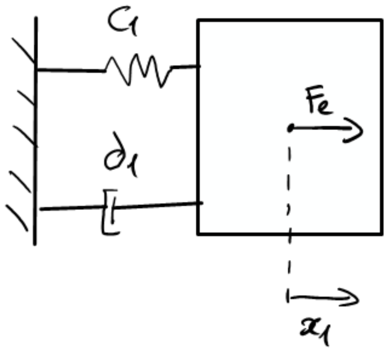
2-DOF Oscillator Results



Co-simulation : Summary of Benefits

- Improve relationships between OEM and Suppliers
  - IP Protection
- Tool interoperability
  - Most current success of co-simulation lies on coupling two different simulation tools.
  - Functional Mockup Interface Standard
- Multi-rate

Example: S1



$$t = nH \rightarrow (n+1)H$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dv_1}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{m_1}c_1 & -\frac{1}{m_1}d_1 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t)$$

$$y_1 = \begin{bmatrix} x_1 \\ v_1 \end{bmatrix}$$

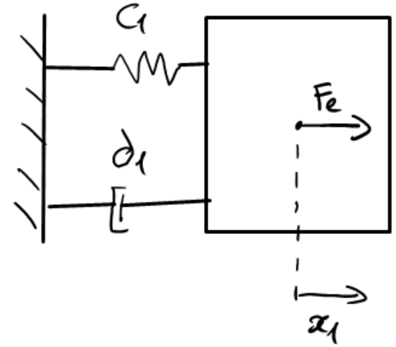
Apply Forward Euler and constant extrapolation of input:

$$k_1 = \frac{H}{h_1} \qquad m \leq k_i \qquad \phi_{u_i}(nH + mh_i) = u_i(nH)$$

$$\begin{bmatrix} x_1(nH + (m+1)h_1) \\ v_1(nH + (m+1)h_1) \end{bmatrix} = \begin{bmatrix} 1 & h_1 \\ -h_1\frac{1}{m_1}c_1 & 1 - h_1\frac{1}{m_1}d_1 \end{bmatrix} \begin{bmatrix} x_1(nH + mh_1) \\ v_1(nH + mh_1) \end{bmatrix} + \begin{bmatrix} 0 \\ h_1 \end{bmatrix} u_1(nH)$$

$$\begin{bmatrix} x_1^{(m+1)} \\ v_1^{(m+1)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & h_1 \\ -h_1\frac{1}{m_1}c_1 & 1 - h_1\frac{1}{m_1}d_1 \end{bmatrix}}_{A_1} \begin{bmatrix} x_1^{(m)} \\ v_1^{(m)} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ h_1 \end{bmatrix}}_{B_1} u_1^{(0)}$$

Example: S1



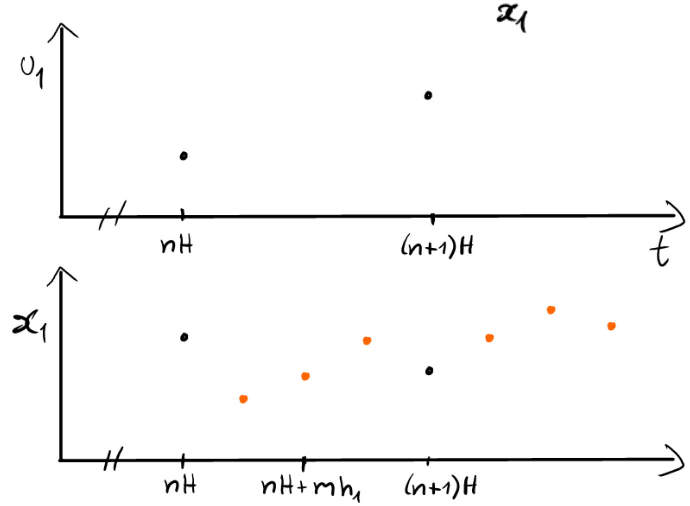
$$t = nH \rightarrow (n+1)H$$

$$k_1 = \frac{H}{h_1} \qquad m \leq k_i$$

$$\begin{bmatrix} x_1^{(m+1)} \\ v_1^{(m+1)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & h_1 \\ -h_1 \frac{1}{m_1} c_1 & 1 - h_1 \frac{1}{m_1} d_1 \end{bmatrix}}_{A_1} \begin{bmatrix} x_1^{(m)} \\ v_1^{(m)} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ h_1 \end{bmatrix}}_{B_1} u_1^{(0)}$$

$$\begin{bmatrix} x_1^{(k_1)} \\ v_1^{(k_1)} \end{bmatrix} = A_1^{k_1} \begin{bmatrix} x_1^{(0)} \\ v_1^{(0)} \end{bmatrix} + \left(\sum_{m=0}^{k_1 - 1} A_1^m B_1 \right) u_1^{(0)}$$

$$\begin{bmatrix} x_1((n+1)H) \\ v_1((n+1)H) \end{bmatrix} = A_1^{k_1} \begin{bmatrix} x_1(nH) \\ v_1(nH) \end{bmatrix} + \left(\sum_{m=0}^{k_1 - 1} A_1^m B_1 \right) u_1(nH)$$

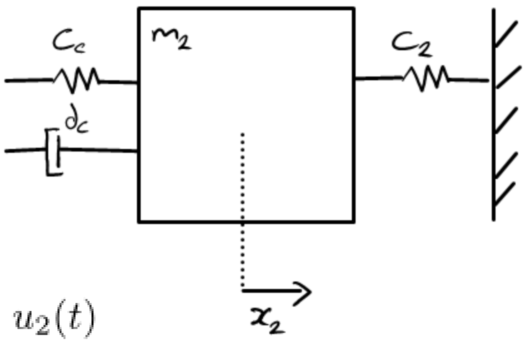


Co-simulation Step:

$$\begin{bmatrix} x_1^{(n+1)} \\ v_1^{(n+1)} \end{bmatrix} = A_1^{k_1} \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \end{bmatrix} + \left(\sum_{m=0}^{k_1 - 1} A_1^m B_1 \right) u_1^{(n)}$$

$$y_1^{(n)} = \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \end{bmatrix}$$

Example: S2



$$t = nH \to (n+1)H$$

$$\begin{bmatrix} \frac{dx_2}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{m_2}(c_2 + c_k) & -\frac{1}{m_2}(d_2 + d_k) \end{bmatrix} \begin{bmatrix} x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_2} c_k & \frac{1}{m_2} d_k \end{bmatrix} u_2(t)$$

$$y_2 = \begin{bmatrix} c_k & d_k \end{bmatrix} \begin{bmatrix} x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} -c_k & -d_k \end{bmatrix} u_2(t)$$

Apply Forward Euler and constant extrapolation of input:

$$\begin{bmatrix} x_2^{(m+1)} \\ v_2^{(m+1)} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & h_2 \\ -h_2 \frac{1}{m_2}(c_2 + c_k) & 1 - h_2 \frac{1}{m_2}(d_2 + d_k) \end{bmatrix}}_{A_2} \begin{bmatrix} x_2^{(m)} \\ v_2^{(m)} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ h_2 \frac{1}{m_2} c_k & h_2 \frac{1}{m_2} d_k \end{bmatrix}}_{B_2} u_2^{(0)}$$

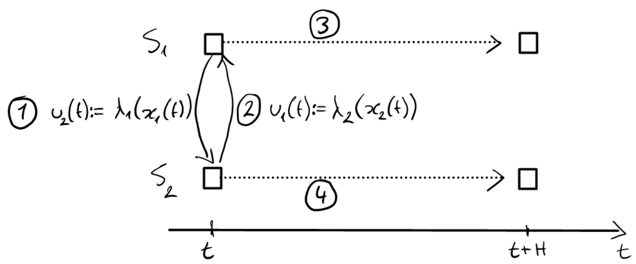
Co-simulation Step:

$$\begin{bmatrix} x_2^{(n+1)} \\ v_2^{(n+1)} \end{bmatrix} = A_2^{k_2} \begin{bmatrix} x_2^{(n)} \\ v_2^{(n)} \end{bmatrix} + \left(\sum_{m=0}^{k_2 - 1} A_2^m B_2 \right) u_2^{(n)}$$

$$y_2^{(n)} = \begin{bmatrix} c_k & d_k \end{bmatrix} \begin{bmatrix} x_2^{(n)} \\ v_2^{(n)} \end{bmatrix} + \begin{bmatrix} -c_k & -d_k \end{bmatrix} u_2^{(n)}$$

Example: 2-DOF Oscillator

$$u_1(t) = y_2(t)$$
$$u_2(t) = y_1(t)$$

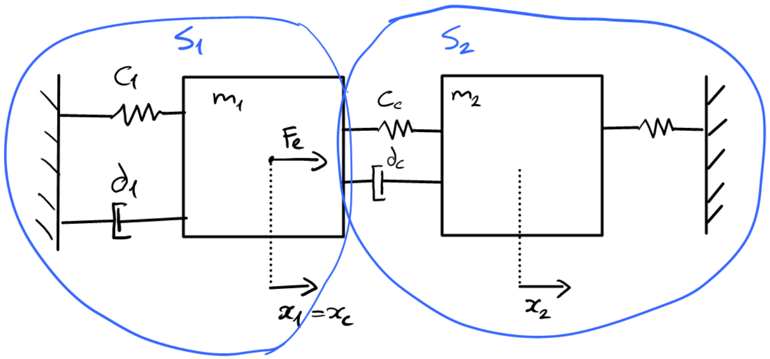


$$\begin{bmatrix} x_1^{(n+1)} \\ v_1^{(n+1)} \end{bmatrix} = A_1^{k_1} \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \end{bmatrix} + \left(\sum_{m=0}^{k_1-1} A_1^m B_1 \right) u_1^{(n)}$$

$$y_1^{(n)} = \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \end{bmatrix}$$

$$\begin{bmatrix} x_2^{(n+1)} \\ v_2^{(n+1)} \end{bmatrix} = A_2^{k_2} \begin{bmatrix} x_2^{(n)} \\ v_2^{(n)} \end{bmatrix} + \left(\sum_{m=0}^{k_2-1} A_2^m B_2 \right) u_2^{(n)}$$

$$y_2^{(n)} = \begin{bmatrix} c_k & d_k \end{bmatrix} \begin{bmatrix} x_2^{(n)} \\ v_2^{(n)} \end{bmatrix} + \begin{bmatrix} -c_k & -d_k \end{bmatrix} u_2^{(n)}$$



Co-simulator:

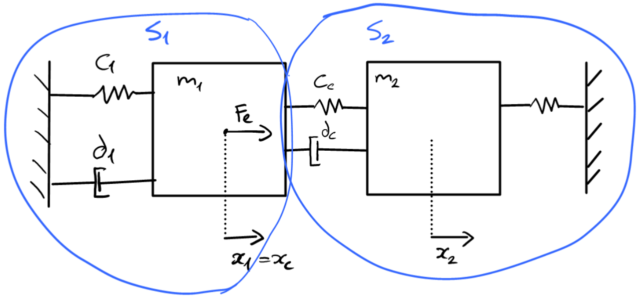
$$\begin{bmatrix} x_1^{(n+1)} \\ v_1^{(n+1)} \\ x_2^{(n+1)} \\ v_2^{(n+1)} \end{bmatrix} = A_{\text{euler}} \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \\ x_2^{(n)} \\ v_2^{(n)} \end{bmatrix}$$

$$A_{\text{euler}} = \begin{bmatrix} A_1^{k_1} & \bar{0} \\ \bar{0} & A_2^{k_2} \end{bmatrix} + \begin{bmatrix} \sum_{m=0}^{k_1-1} A_1^m & \bar{0} \\ \bar{0} & \sum_{m=0}^{k_2-1} A_2^m \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ -h_1 c_k & -h_1 d_k & h_1 c_k & h_1 d_k \\ 0 & 0 & 0 & 0 \\ h_2 \frac{1}{m_2} c_k & h_2 \frac{1}{m_2} d_k & 0 & 0 \end{bmatrix}$$

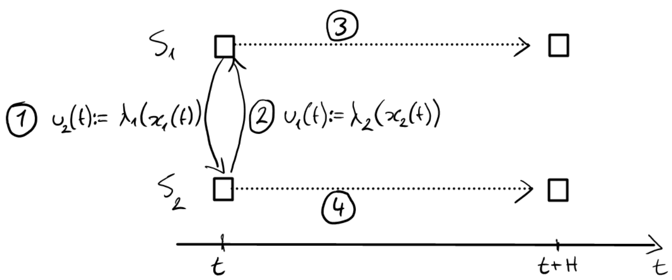
Stability: $\rho(A_{\text{euler}}) < 1$

$$\begin{bmatrix} x_1^{(n+1)} \\ v_1^{(n+1)} \\ x_2^{(n+1)} \\ v_2^{(n+1)} \end{bmatrix} = A_{\text{euler}} \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \\ x_2^{(n)} \\ v_2^{(n)} \end{bmatrix}$$

Recap

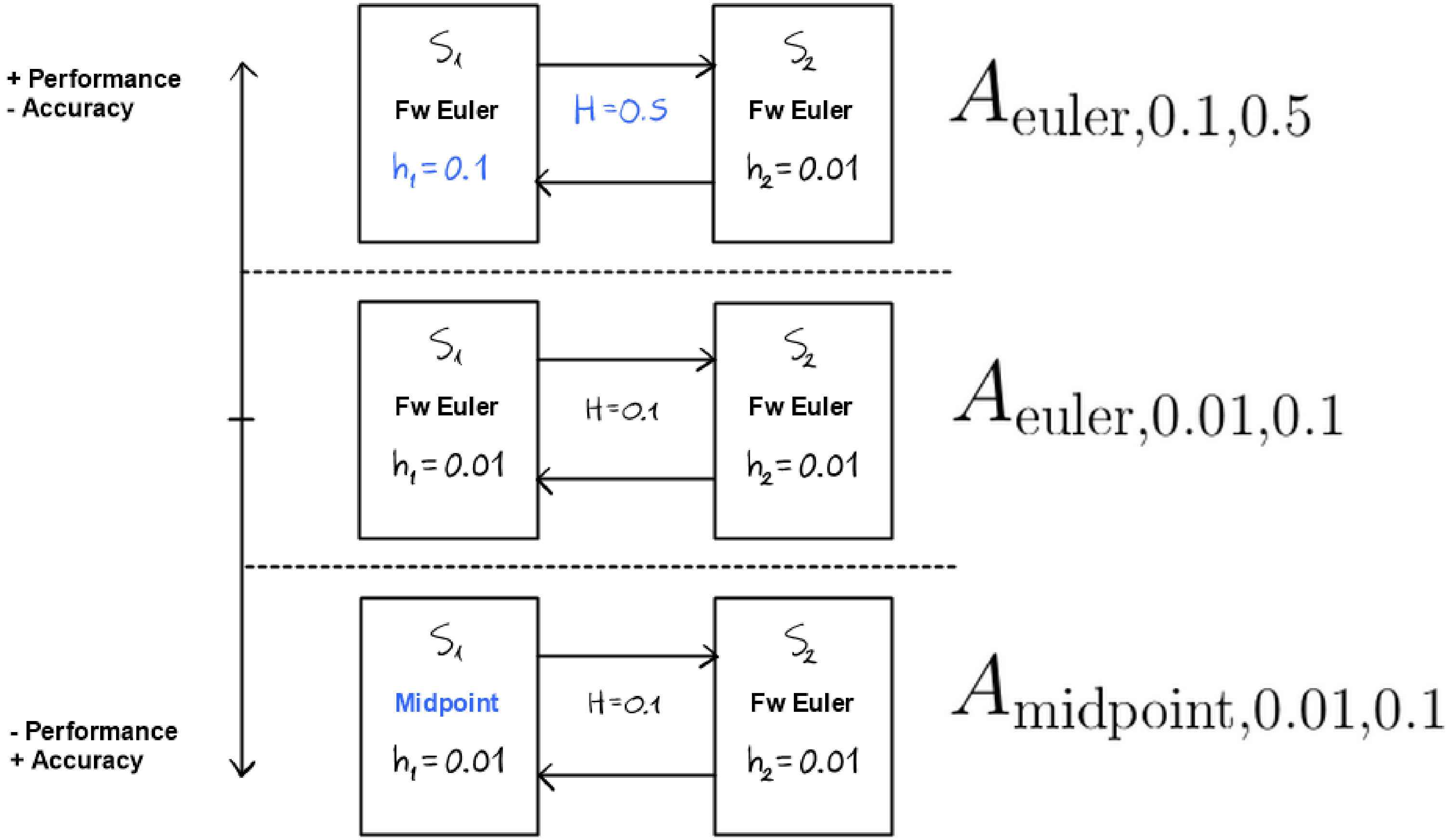


- Co-simulator (Scenario + Orch. Alg.) as a difference equation, function of
  - Coupling approach (we used Jacobi iteration for state and Gauss-Seidel for output propagation)
  - Communication step size H (which influences ki)
  - Simulator's specification:
    - Input extrapolation (we used constant extrapolation)
    - Numerical method (we assumed Forward Euler)
    - Micro-step size hi (which influences ki)
    - Model (level of abstraction, degrees-of-freedom)



$$A_{\text{euler}} = \begin{bmatrix} A_1^{k_1} & \bar{0} \\ \bar{0} & A_2^{k_2} \end{bmatrix} + \begin{bmatrix} \sum_{m=0}^{k_1-1} A_1^m & \bar{0} \\ \bar{0} & \sum_{m=0}^{k_2-1} A_2^m \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ -h_1 c_k & -h_1 d_k & h_1 c_k & h_1 d_k \\ 0 & 0 & 0 & 0 \\ h_2 \frac{1}{m_2} c_k & h_2 \frac{1}{m_2} d_k & 0 & 0 \end{bmatrix}$$

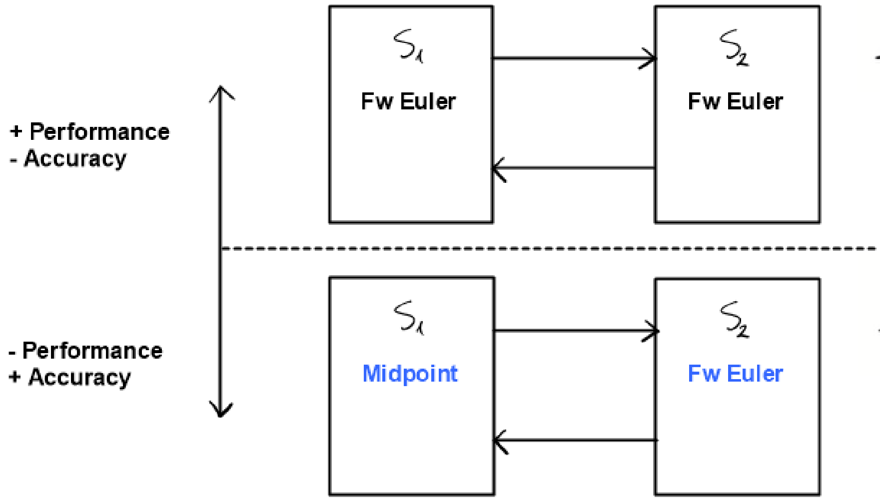
Co-simulation as a Switched System



Example: Stabilized Switched System

$h_1 = 0.04$
$k_1 = 5$
$h_2 = 0.2$
$k_2 = 1$
$H = 0.2$

$m_1 = 1$
$m_2 = 1$
$c_1 = 1$
$c_2 = 1$
$d_1 = 0.1$
$d_2 = 0.1$
$c_k = 0.1$
$d_k = 1$



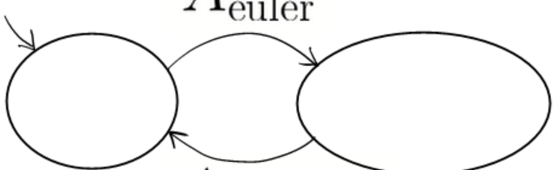
A_{euler}

$\rho(A_{\text{euler}}) \approx 1.002317120986636 > 1$

A_{midpoint}

$\rho(A_{\text{midpoint}}) \approx 0.997568283024404 < 1$

$h_1 = 0.01$
$k_1 = 10$
$h_2 = 0.1$
$k_2 = 1$
$H = 0.1$



$\rho(A_{\text{midpoint}} A_{\text{euler}}) \approx 0.999863522587465 < 1$

$$\begin{bmatrix} x_1^{(n+2)} \\ v_1^{(n+2)} \\ x_2^{(n+2)} \\ v_2^{(n+2)} \end{bmatrix} = A_{\text{midpoint}} A_{\text{euler}} \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \\ x_2^{(n)} \\ v_2^{(n)} \end{bmatrix}$$

$$\frac{1}{2}\left(\underbrace{\frac{t_f}{0.2}(5 \times 1 + 1 \times 1)}_{A_{\text{euler}}} + \underbrace{\frac{t_f}{0.1}(5 \times 1 + 1 \times 1)}_{A_{\text{midpoint}}} \right)$$

General Idea

- Take multiple simulators for the same subsystem
  - At different levels of abstraction
    - Model Order Reduction
    - Phase space quantization (conversion of equations to state automata)
  - With different specifications (solver, micro-step sizes, extrapolations, etc…)
- and find a constrained switching system such that:
  - Co-simulator is always stable for valid switch signals, and
  - Optimality criteria (e.g., meet real time deadlines) is met

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Roadmap on Optimization of Co-simulation

- Real-time co-simulation case study
  - Fixed step size, performance/accuracy tradeoff
- Address black box linear components
  - Linear system identification
- Non-linear simulators
- Scalability
  - Co-simulation scenarios with 700 simulators
  - 10-100 DOF
- Your ideas ☺