Co-simulation: State of the art

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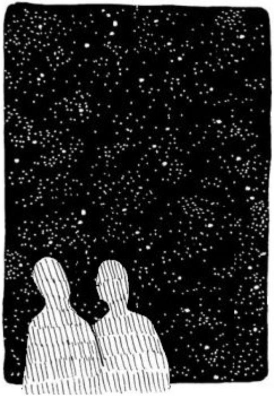
Why co-simulation?

And why are we here?



Motivation(s) for Co-simulation

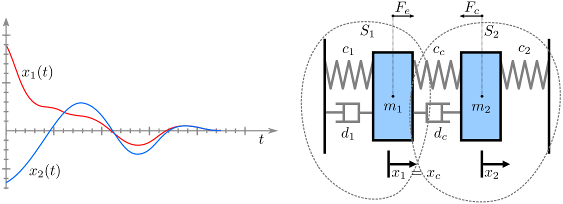
Definition: Simulation of a coupled system,
via the composition of sub-system simulations.

Main reasons:

- Performance/Accuracy;
- Heterogeneity of languages and tools;
- Intellectual Property protection;

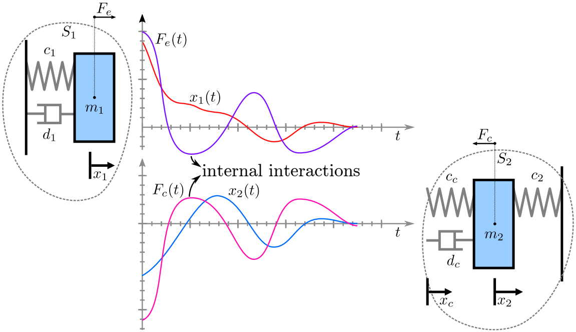
Main goal: unlock the full potential of simulation.

Why are we here?



Simulation of a coupled system...

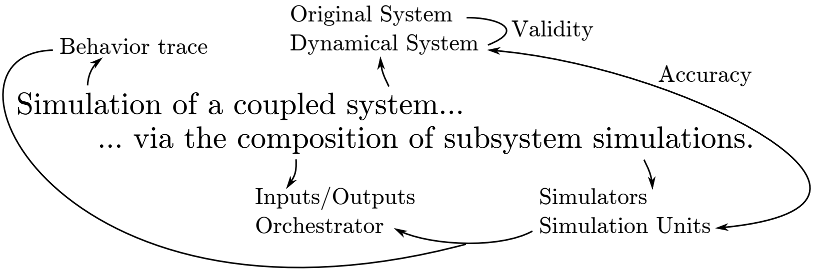
... via the composition of subsystem simulations.



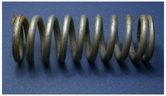
Outline

- Terminology
- Simulation units
- Input extrapolation techniques
- Orchestration algorithms
- Algebraic loops
- Convergence
- Stability
- Wrap-up
- Ongoing work

Background Overview



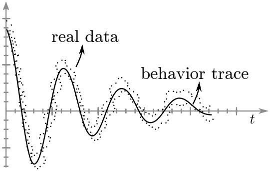
Dynamical Systems

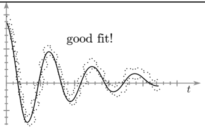
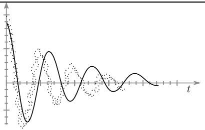
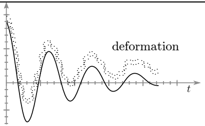


evolution state inputs

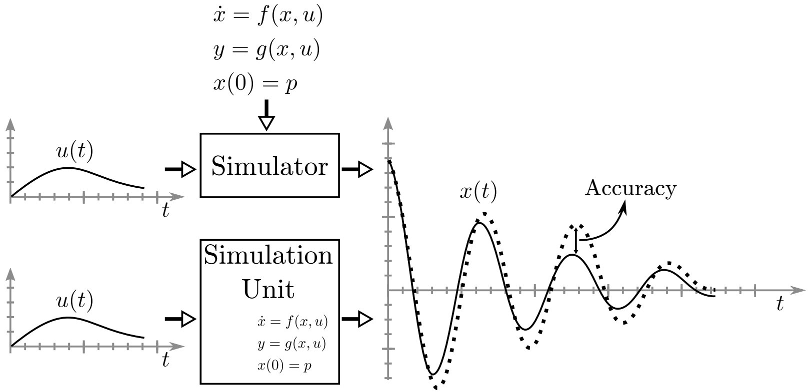
$$\dot{x} = f(x, u)$$
$$y = g(x, u)$$
$$x(0) = p$$

outputs



	Experimental Frame	Validity
Valid model	✓	
Invalid model	✓	
N/A	×	

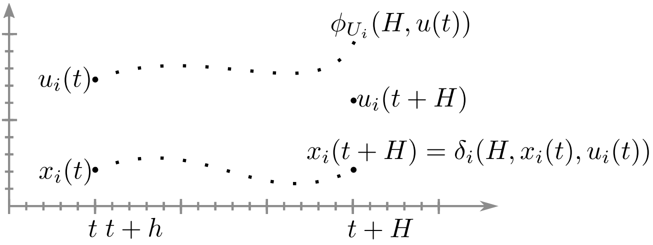
Simulators



Correct SU = Accurate Simulator + Valid Model

Simulation Unit

$$S_i = \langle X_i, U_i, Y_i, \delta_i, \lambda_i, x_i(0), \phi_{U_i} \rangle$$
$$\delta_i : \mathbb{R} \times X_i \times U_i \to X_i$$
$$\lambda_i : \mathbb{R} \times X_i \times U_i \to Y_i \text{ or } \mathbb{R} \times X_i \to Y_i$$
$$x_i(0) \in X_i$$
$$\phi_{U_i} : \mathbb{R} \times U_i \times \ldots \times U_i \to U_i$$



Simulation Unit and the Functional Mockup Unit

$$S_i = \langle X_i, U_i, Y_i, \delta_i, \lambda_i, x_i(0), \phi_{U_i} \rangle$$

$$y_i := \lambda_i(t, x_i, u_i);$$
$$x_i := \delta_i(H, x_i, u_i);$$

$\text{fmi2SetFMUstate}_i(c_i, x_i);$
$\text{fmi2SetReal}_i(c_i, \ldots, \dim(U_i), u_i);$
$\text{fmi2GetReal}_i(c_i, \ldots, \dim(Y_i), y_i);$
$\text{fmi2DoStep}_i(c_i, t, H, \ldots);$
$\text{fmi2GetFMUstate}_i(c_i, \&x_i);$

$$x_i := \delta_i(H, x_i, u_i);$$
$$y_i := \lambda_i(t + H, x_i, u_i);$$

$\text{fmi2SetFMUstate}_i(c_i, x_i);$
$\text{fmi2SetReal}_i(c_i, \ldots, \dim(U_i), u_i);$
$\text{fmi2DoStep}_i(c_i, t, H, \ldots);$
$\text{fmi2GetFMUstate}_i(c_i, \&x_i);$
$\text{fmi2SetReal}_i(c_i, \ldots, \dim(U_i), u_i);$
$\text{fmi2GetReal}_i(c_i, \ldots, \dim(Y_i), y_i);$

Types of Simulation Units

$$S_i = \langle X_i, U_i, Y_i, \delta_i, \lambda_i, x_i(0), \phi_{U_i} \rangle$$

State transition:

reactive $x_i(t+H) = \delta_i(H, x_i(t), \mathbf{u_i}(\mathbf{t}+\mathbf{H}))$

delayed $x_i(t+H) = \delta_i(H, x_i(t), \mathbf{u_i}(\mathbf{t}))$

Output:

mealy $y_i(t) = \lambda_i(t, x_i(t), \mathbf{u_i}(\mathbf{t}))$

moore $y_i(t) = \lambda_i(t, x_i(t))$

Types of Input Extrapolations

Constant

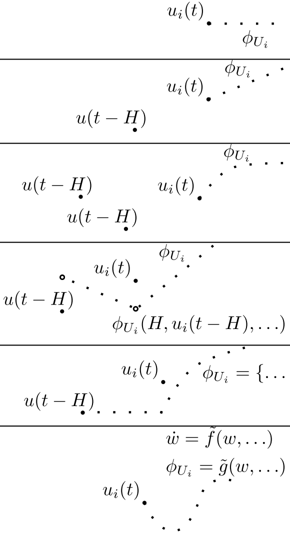
Linear

Polynomial

Extrapolated/Interpolation

Context-aware

Model ID'ed



Checkpoint

- Dynamical Systems
- Simulators
- Simulation units (externals and internals)

- Interactions between Simulation Units

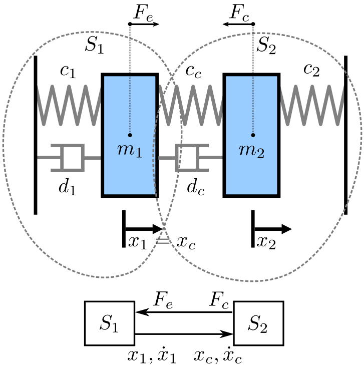
Co-simulation Scenario

$$\langle \{S_i : i \in D\}, L \rangle$$
$$S_i = \langle X_i, U_i, Y_i, \delta_i, \lambda_i, x_i(0), \phi_{U_i} \rangle$$
$$L : (\Pi_{i \in D} Y_i) \times Y_{CS} \times (\Pi_{i \in D} U_i) \times U_{CS} \to \mathbb{R}^m$$

Coupling: $L = \bar{0}$

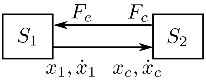
$$\langle \{1, 2\}, \{S_1, S_2\}, L \rangle$$

$$L = \begin{bmatrix} x_c - v_1 \\ \dot{x}_c - x_1 \\ F_e - F_c \end{bmatrix}$$



Jacobi Type Orchestrator

$$\langle \{S_i : i \in D\}, L\rangle$$
$$S_i = \langle X_i, U_i, Y_i, \delta_i, \lambda_i, x_i(0), \phi_{U_i}\rangle$$
$$L : (\Pi_{i\in D} Y_i) \times Y_{CS} \times (\Pi_{i\in D} U_i) \times U_{CS} \to \mathbb{R}^m$$



$t := 0$;
$x_i := x_i(0)$ for $i = 1, \ldots, n$;
while *true* **do**

Solve the following system for the unknowns:

$$\begin{cases} y_1 = \lambda_1(t, x_1, u_1) \\ \ldots \\ y_n = \lambda_n(t, x_n, u_n) \\ L(y_1, \ldots, y_n, y_{CS}, u_1, \ldots, u_n) = \bar{0} \end{cases}$$

$x_i := \delta_i(H, x_i, u_i)$, for $i = 1, \ldots, n$;
$t := t + H$

end

* Delayed units only.

Compositional Co-simulation

The *raison d'etre* of the orchestrator is to produce a correct co-simulation trace, assuming that *each simulation unit is correct*.

Algebraic Loops

I/O

x_i, x_j known.

$$\mathbf{y_i} = \lambda_i(t, x_i, \mathbf{u_i})$$
$$\mathbf{u_j} = \mathbf{y_i}$$
$$\mathbf{y_j} = \lambda_j(t, x_j, \mathbf{u_j})$$
$$\mathbf{u_i} = \mathbf{y_j}$$

State and I/O

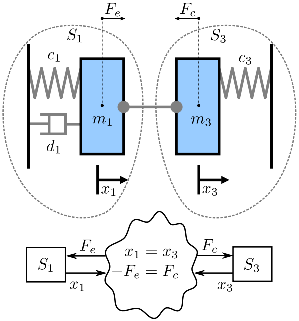
x_i, x_j, y_j, u_i known.

$$\tilde{\mathbf{x}}_\mathbf{i} = \delta_i(H, x_i, u_i)$$
$$\tilde{\mathbf{y}}_\mathbf{i} = \lambda_i(t + H, \tilde{\mathbf{x}}_\mathbf{i}, \tilde{\mathbf{u}}_\mathbf{i})$$
$$\tilde{\mathbf{u}}_\mathbf{j} = \tilde{\mathbf{y}}_\mathbf{i}$$
$$\tilde{\mathbf{x}}_\mathbf{j} = \delta_j(H, x_j, \tilde{\mathbf{u}}_\mathbf{j})$$
$$\tilde{\mathbf{y}}_\mathbf{j} = \lambda_j(t + H, \tilde{\mathbf{x}}_\mathbf{j})$$
$$\tilde{\mathbf{u}}_\mathbf{i} = \tilde{\mathbf{y}}_\mathbf{j}$$

Fixed point iterations

Strong coupling, Waveform iteration, Semi-implicit

Algebraic Couplings



$$g(F_e) = \tilde{x}_1(F_e) - \tilde{x}_3(-F_e) = 0$$

1 Guess F_e;

2 $\hat{x}_1 := \delta_1(H, x_1, F_e)$;

3 $\hat{x}_3 := \delta_i(H, x_3, -F_e)$;

4 **if** $\hat{x}_1 \approx \hat{x}_3$ **then**

5   Done;

6 **end**

7 $\bar{x}_1 := \delta_1(H, x_1, F_e + \epsilon)$;

8 $\bar{x}_3 := \delta_i(H, x_3, -F_e + \epsilon)$;

9 $\frac{\partial x_1}{\partial F_e} \approx \frac{\bar{x}_1 - \hat{x}_1}{\epsilon}$;

10 $\frac{\partial x_2}{\partial F_c} \approx \frac{\bar{x}_3 - \hat{x}_3}{\epsilon}$;

11 $\frac{\partial g}{\partial F_e} = \frac{\partial x_1}{\partial F_e} + \frac{\partial x_2}{\partial F_c}$;

12 $F_e := F_e(n \cdot H) - \left[\frac{\partial g(F_e(n \cdot H))}{\partial F_e}\right]^{-1} \cdot g(F_e(n \cdot H))$;

13 Go to Line 1;

Error Control

- Convergence – Deviation of co-simulation trace from true solution $e(t)$ *ultimately* $(t \to 0)$ tends to zero, as $H \to 0$.
  - Sufficient condition: coupled model is an ODE.
  - Danger: algebraic loops *in the coupled model*.
  - Order bottle neck is ϕ_{U_i}.
- Error Estimation
  - Richardson extrapolation: compare steps with half-steps;
  - Multi-Order Input Extrapolation: compare different order input approximations;
  - Milne's Device: compare guessed input with given input;
  - Parallel Embedded Method: take derivative of some unit and run an ODE solver in parallel;
  - Conservation Laws: track energy excesses/defects;
  - Embedded Solver Method: let units decide;
- Step size selection: all traditional simulation techniques apply.

Stability

Relevant question: does the orchestrator cause $\lim_{t\to\infty} e(t) \neq 0$, for $H > 0$?

- Assume coupled ODE system (which must be LTI) is stable: $\lim_{t\to\infty} \hat{x}(t) = 0$
- Write each simulation unit as a discrete time system: $x_i^{(n+1)} = e^{A_i H} x_i^{(n)} + K_i B_i u_i^{(n)}$
- And its output: $y_i^{(n+1)} = C_i e^{A_i H} x_i^{(n)} + (C_i K_i B_i\, [+D_i]) u_i^{(n)}$
- Replacing all inputs u_i by the coupling conditions, we get a big discrete system:

$$
\begin{bmatrix} \begin{bmatrix} x_1^{(n+1)} \\ v_1^{(n+1)} \end{bmatrix} \\ y_1^{(n+1)} \\ \begin{bmatrix} x_2^{(n+1)} \\ v_2^{(n+1)} \end{bmatrix} \\ y_2^{(n+1)} \end{bmatrix} = \underbrace{\begin{bmatrix} e^{A_1 H} & \bar{0} & \bar{0} & K_1 B_1 \\ C_1 e^{A_1 H} & \bar{0} & \bar{0} & C_1 K_1 B_1 \\ \bar{0} & K_2 B_2 & e^{A_2 H} & \bar{0} \\ \bar{0} & C_2 K_2 B_2 + D_2 & C_2 e^{A_2 H} & \bar{0} \end{bmatrix}}_{A} \begin{bmatrix} \begin{bmatrix} x_1^{(n)} \\ v_1^{(n)} \end{bmatrix} \\ y_1^{(n)} \\ \begin{bmatrix} x_2^{(n)} \\ v_2^{(n)} \end{bmatrix} \\ y_2^{(n)} \end{bmatrix}
$$

- Check if $\rho(A) < 1$

Summary

- Simulation units
- Orchestration algorithms
- Compositionality for correct co-simulation
- Threats to compositionality

Thank you!

References

[1] Cláudio Gomes. Foundations for Co-simulation – IWT Proposal. Technical report, University of Antwerp, Antwerp, 2015.

[2] Cláudio Gomes. Foundations for Continuous Time Hierarchical Co-simulation. In *ACM Student Research Competition (ACM/IEEE 19th International Conference on Model Driven Engineering Languages and Systems)*, page to appear, Saint Malo, Brittany, France, 2016.

[3] Cláudio Gomes, Casper Thule, David Broman, Peter Gorm Larsen, and Hans Vangheluwe. Co-simulation: State of the art. Technical report, feb 2017.