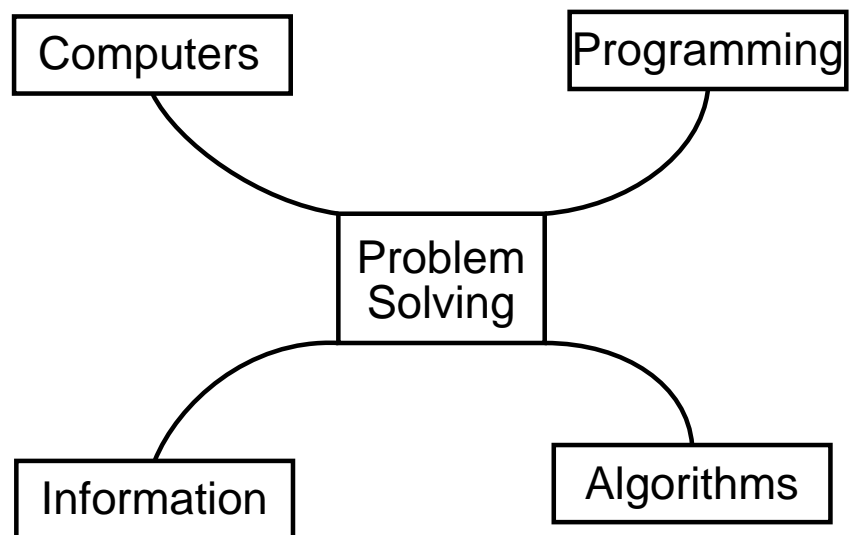
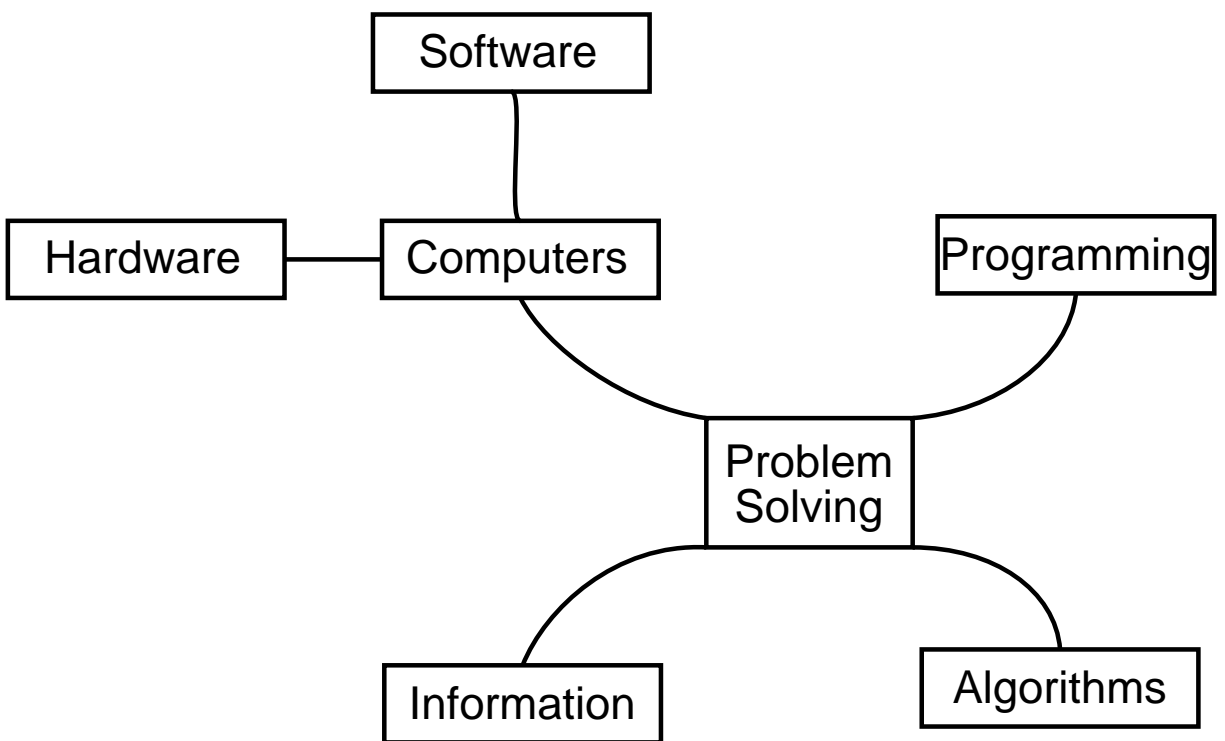

Reminder

- Midterm today at 6:00pm at MAASS 112

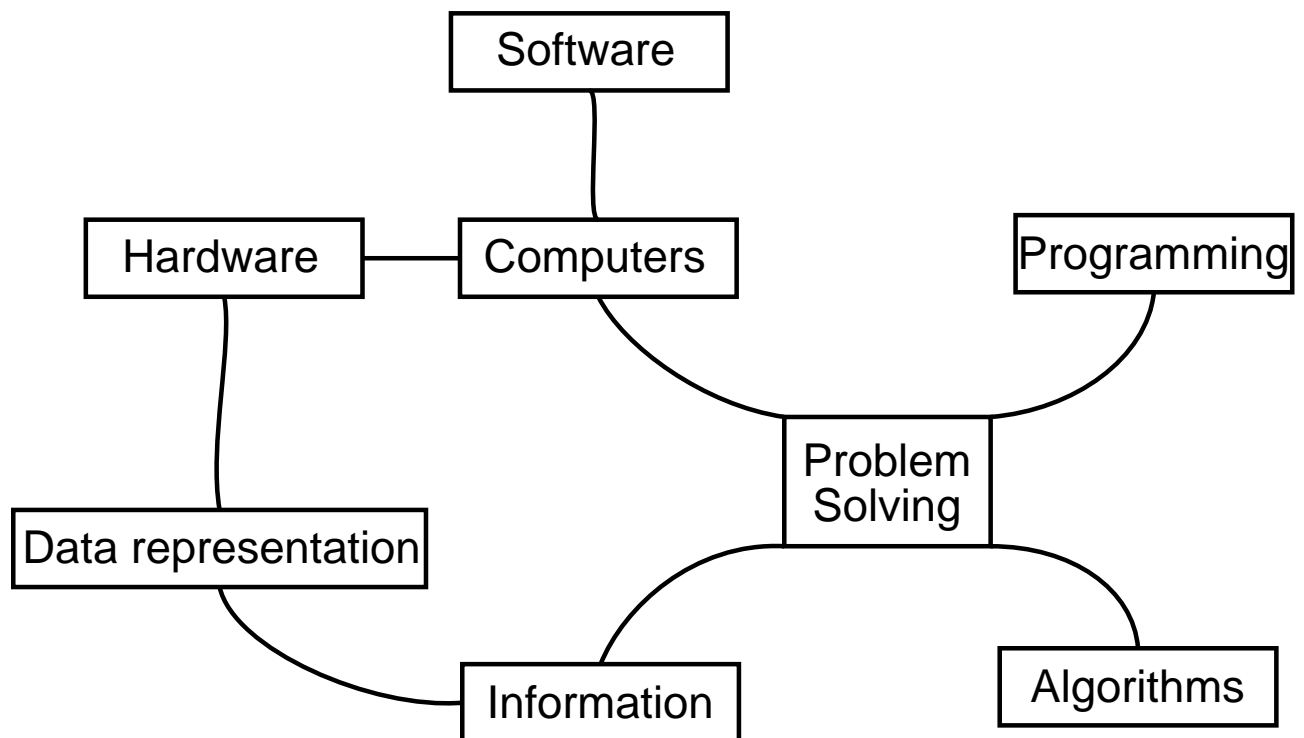
Review



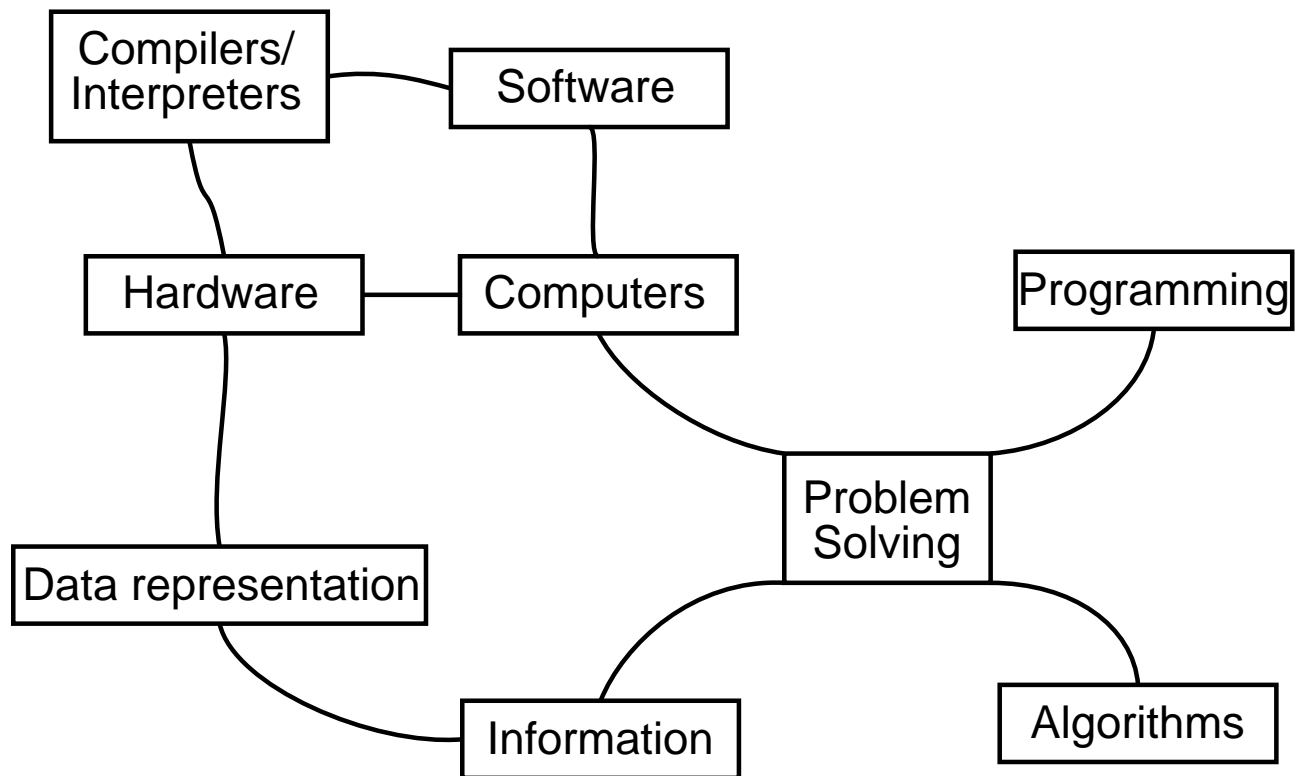
Review



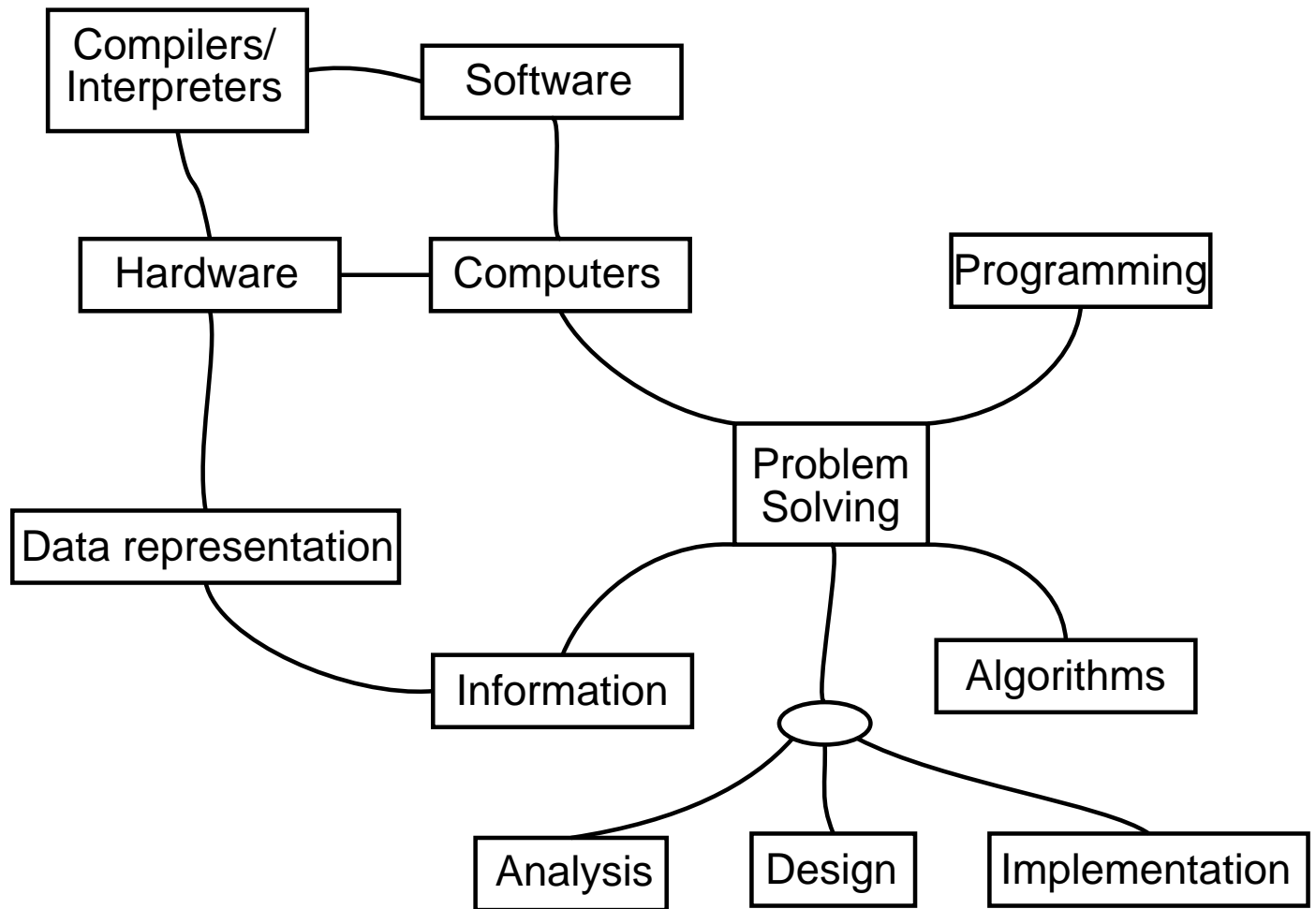
Review



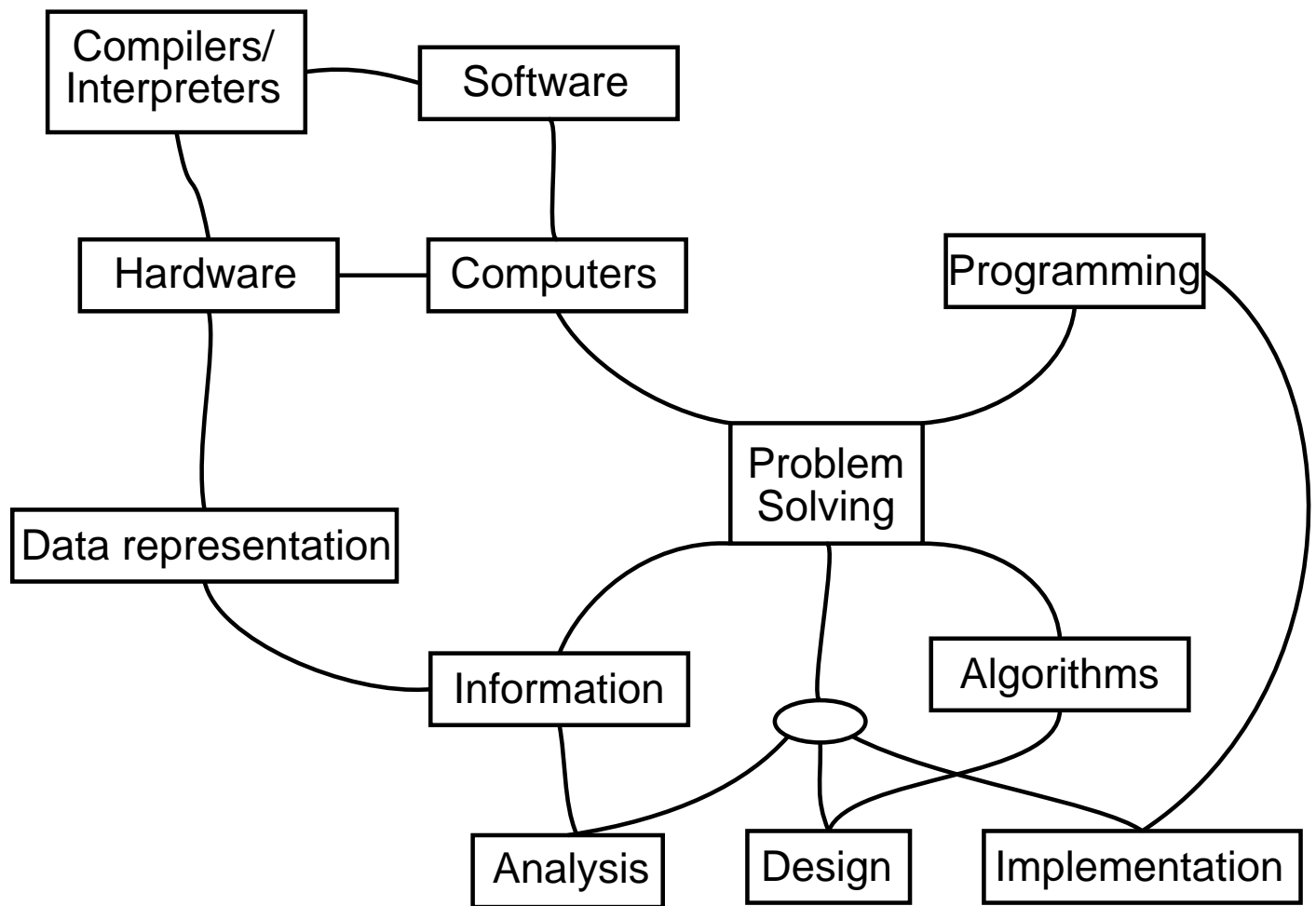
Review



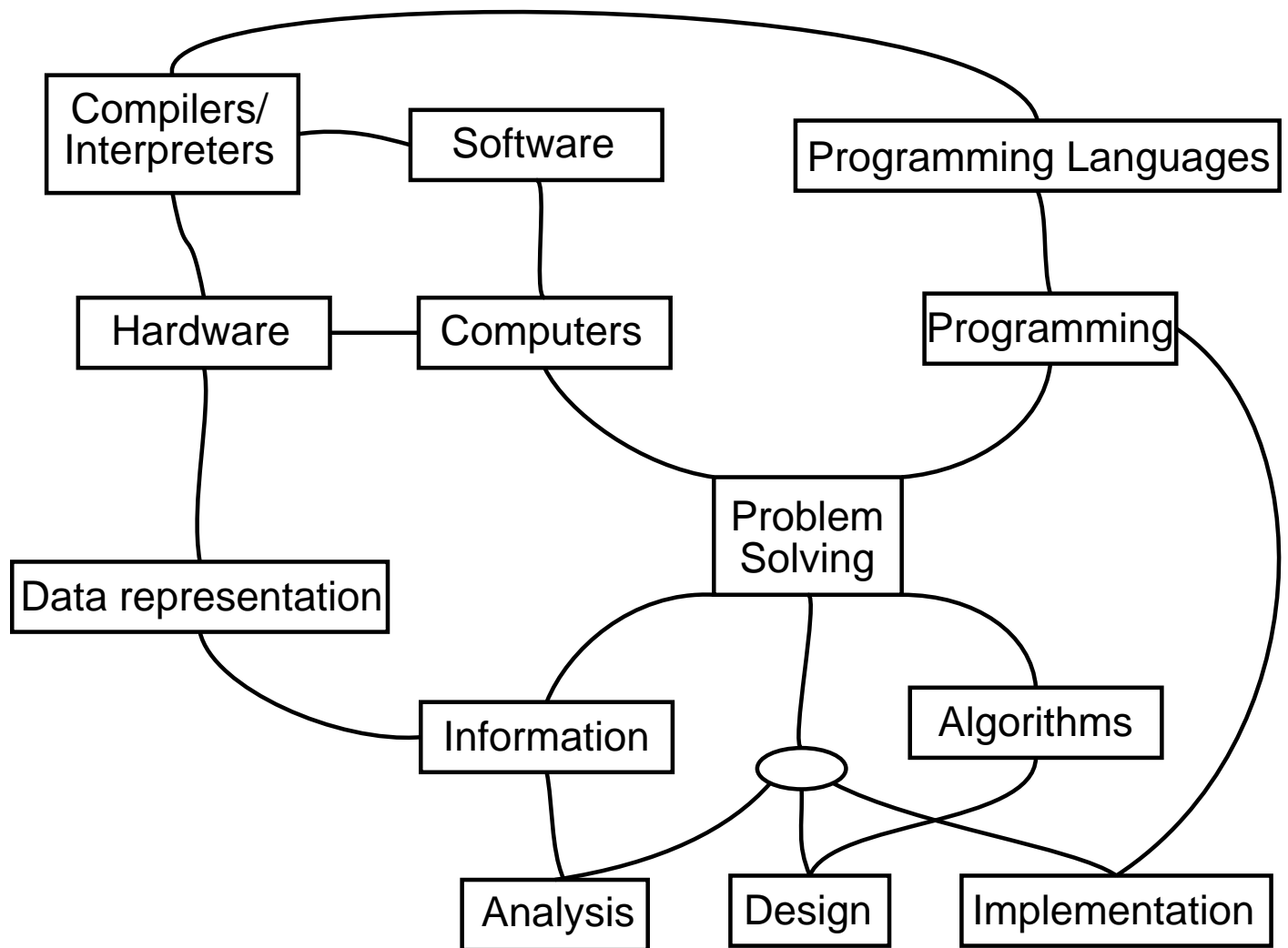
Review



Review



Review



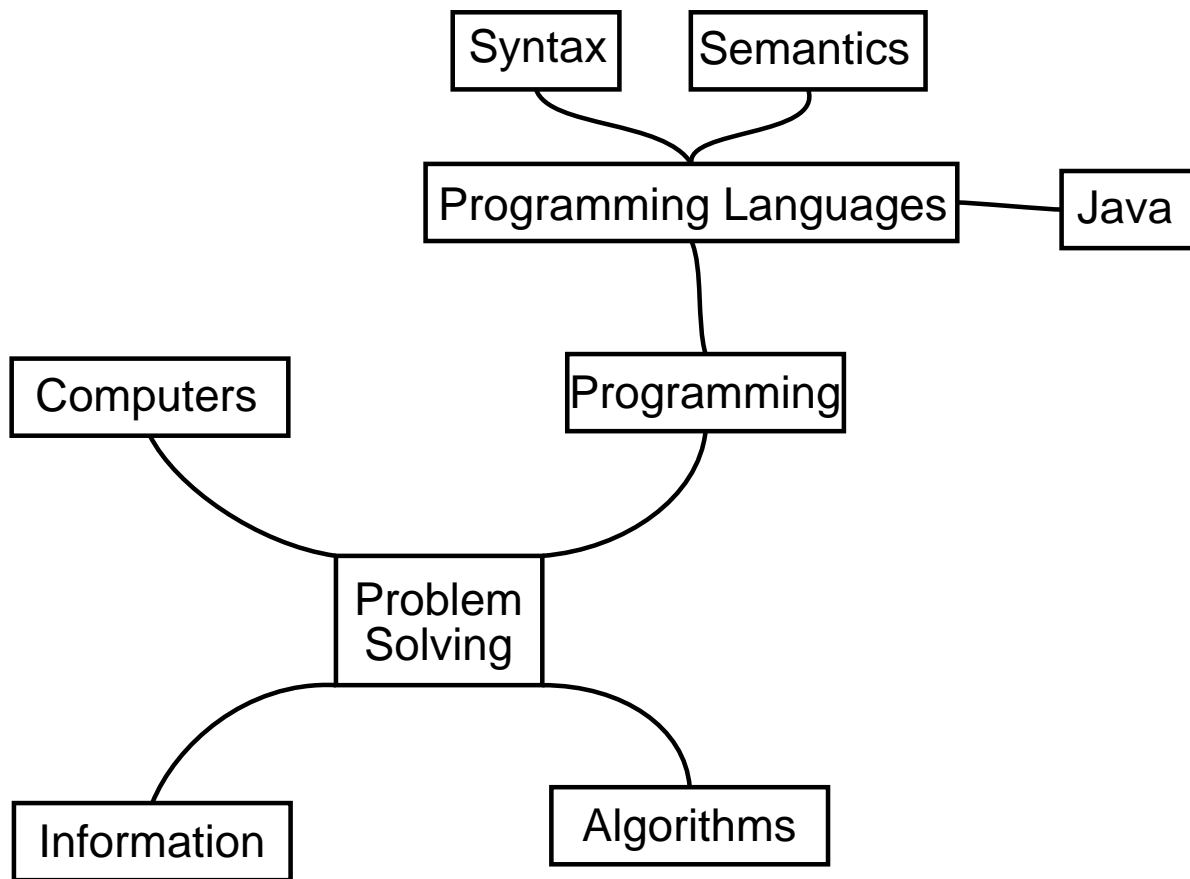
Problem solving

- Clear statement of the problem
- Analysis (of the problem)
- Design
- Implementation
- Testing / Verification
- Maintenance

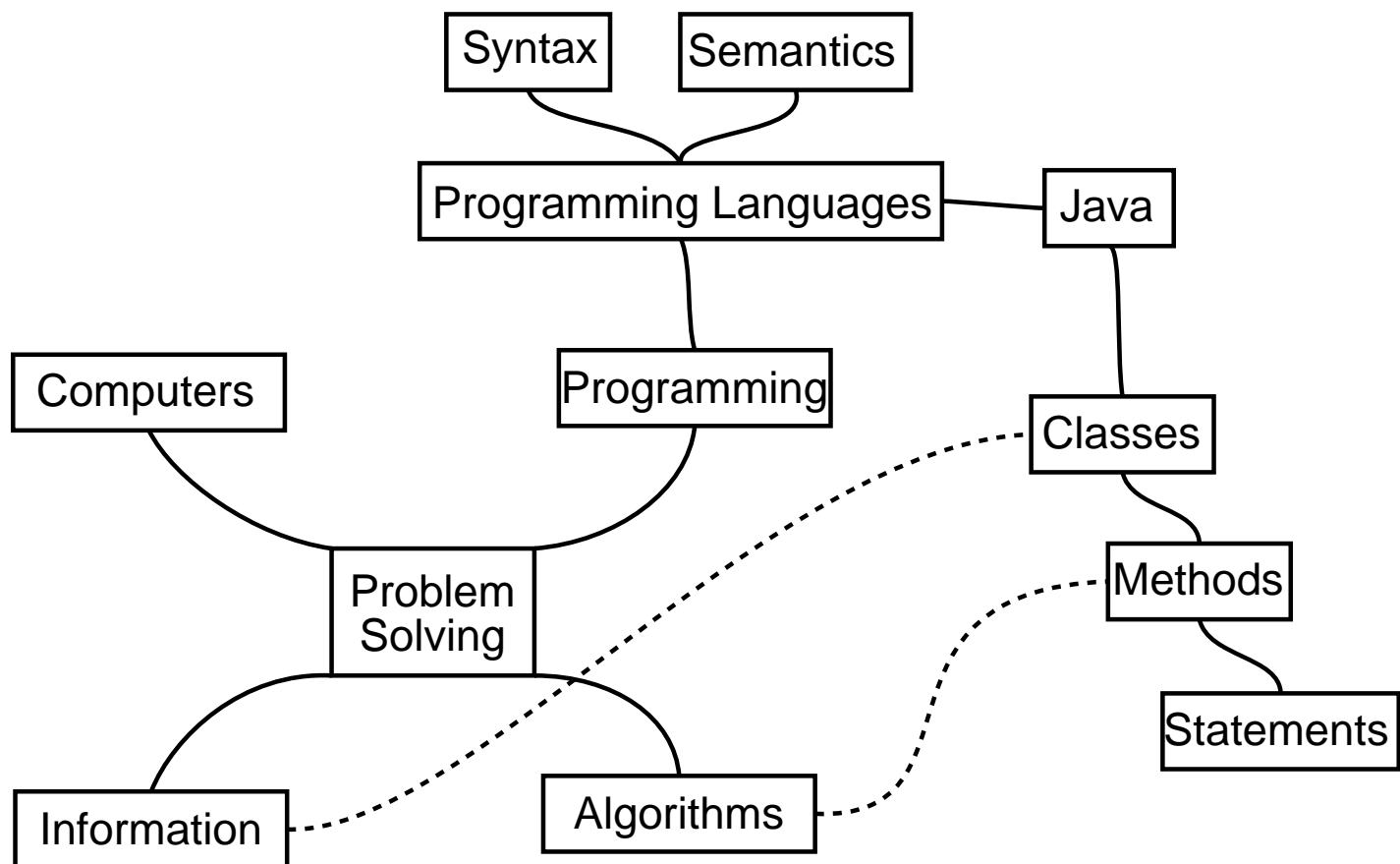
Analysis

- Goal: to obtain a precise understanding the problem
- Things to do in analysis:
 - Determine inputs and outputs
 - Determine general and specific requirements
 - Make or obtain precise definitions of concepts involved
 - Determine the relevant information to the problem
 - Determine the relationship between different elements or pieces of information of the problem
 - Make explicit any relevant assumptions

Review



Review



Statements

- Variable declarations
- Assignment
- Conditionals
- Loops
- Method calls

Statements

- Variables

- A variable is a memory cell with a name (identifier)
- A variable can change its value
- A variable has a *data type*
- Variable declaration

```
type identifier;
```

```
type identifier = expression;
```

```
type id1, id2, id3;
```

```
type id1 = e1, id2 = e2, id3 = e3;
```

- The type of a variable can be
 - * Primitive (e.g. int, boolean, etc.)
 - * User-defined (i.e. a class)

Statements

- Assignment
 - Changes the value of a variable
 - variable = expression;*
 - Expressions:
 - * arithmetic: int type
 - * boolean
 - * string

Statements

- Conditionals: making decisions

- if

```
if (condition)  
    statement_or_block;
```

- if-else

```
if (condition)  
    statement_or_block1;  
else  
    statement_or_block2;
```

- switch

```
switch(int_or_char_expr) {  
    case int_or_char_expr1: statements1;  
    case int_or_char_expr2: statements2;  
    ...  
    default: statementsn;  
}
```

Statements

- Loops: Repetition

- while

```
while (condition)  
    statement_or_block;
```

- do-while

```
do  
    statement_or_block  
while(condition);
```

- for

```
for (init; condition; statement)  
    statement_or_block;
```

Statements

- Method calls

- normal

- objectreference.methodname (arg1, arg2, ..., argn)*

- static

- classname.methodname (arg1, arg2, ..., argn)*

- Messages to self

- methodname (arg1, arg2, ..., argn)*

- this.methodname (arg1, arg2, ..., argn)*

Control-flow

- Statements are executed sequentially
- The order in which statements appear matters

```
int x;  
x = 8;  
x = x + 3;  
if (x >= 10) {  
    System.out.println("OK");  
}  
else {  
    System.out.println("Error");  
}
```

Control-flow

- Statements are executed sequentially
- The order in which statements appear matters

```
int x;  
x = 8;  
if (x >= 10) {  
    System.out.println("OK");  
}  
else {  
    System.out.println("Error");  
}  
x = x + 3;
```

Control-flow

- Loops
- Termination: a loop may fail to terminate

```
int i;  
i = 0;  
while (i < 10) {  
    System.out.print(i);  
}
```

- To terminate, the body of the loop must have some “advance” statements, i.e. statements which will eventually make the condition false.

```
int i;  
i = 0;  
while (i < 10) {  
    System.out.print(i);  
    i++;  
}
```

Classes and objects

- Objects are composite data that can react to messages
- Classes define the structure and behaviour of objects
- Objects are not classes
 - A class is a data type
 - An object is an instance of a class

Classes and objects

- Defining classes
 - Attributes (instance variables): characteristics of the objects
 - Methods: how objects react to messages
- Using classes
 - Creating objects
 - Accessing attributes
 - Invoking (calling) methods: sending messages

Classes and objects

- Defining classes:

```
public class ClassName {  
    // Attribute definitions  
    // Method definitions  
}
```

- Defining attributes:

```
type identifier;
```

Note: different classes can have attributes with the same name

Classes and objects

- Defining methods:

- normal

```
type name(type1 p1, type2 p2, ..., typen pn)
{
    list_of_statements;
}
```

- static

```
static type name(type1 p1, type2 p2, ..., typen
{
    list_of_statements;
}
```

Note: different classes can have methods with the same name

Classes and objects

- Using classes:

- Creating objects:

- ```
type objectreference ;
objectreference = new type (args) ;
```
    - where *type* is a class.

- Accessing attributes:

- ```
objectreference.attributename
```

- only if the attribute is defined in *objectreference*'s class.

- Calling methods:

- ```
objectreference.methodname (arg1, arg2, ..., argn)
```

- only if the method is defined in *objectreference*'s class, and the types of the arguments are the same as the types of the parameters.

---

## Methods calls

- If class A is defined as

```
class A {
 void m() { ... }
}
```

- and a variable x has a reference to an A object:

```
A x;
x = new A();
```

- then method m can be applied to x

```
x.m();
```

- But if method p is not defined in A, it cannot be applied to x

```
x.p(); // Error
```

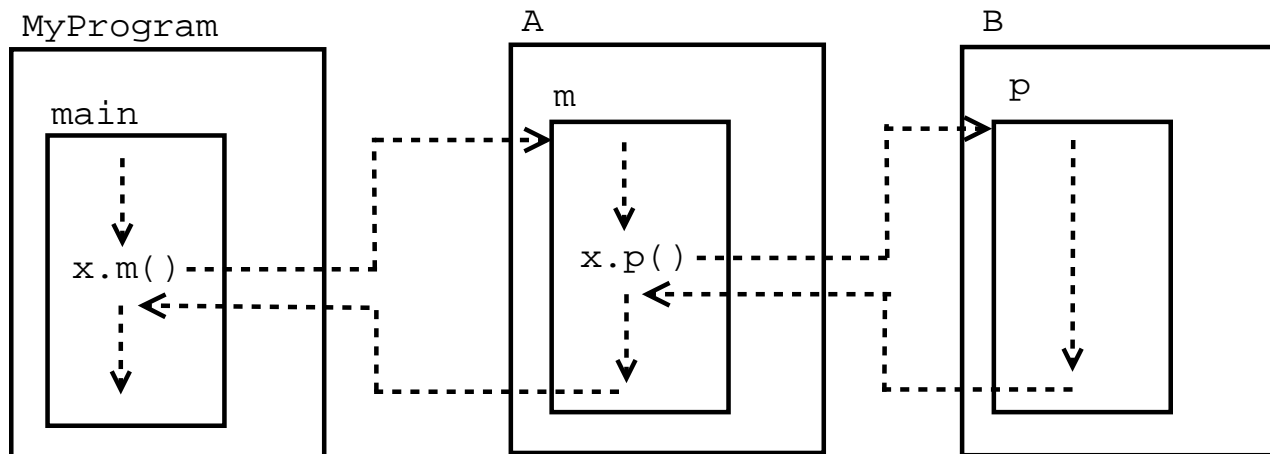
---

## Method calls

- Control flow
- Frames and parameter passing
- Access to variables
  - Parameters
  - Local variables
  - Object attributes
- Static methods
- Static variables
- Recursive methods

---

# Methods calls: control flow



---

## Methods calls: parameter passing

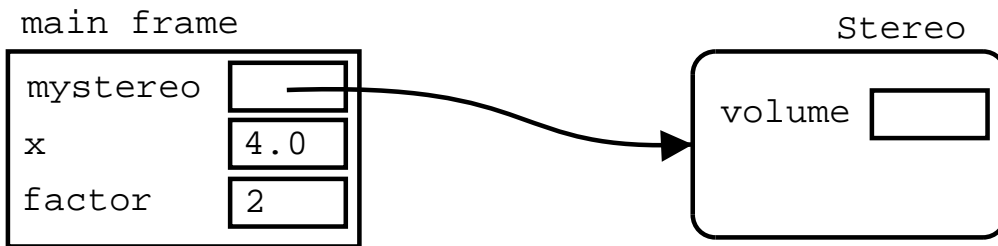
```
public class Stereo {
 double volume;
 void set_volume(double v)
 {
 volume = v;
 }
 double get_volume()
 {
 return volume;
 }
}

public class SoundSystem {
 public static void main(String[] args)
 {
 Stereo mystereo = new Stereo();
 double x, factor = 2;
 System.out.println("Testing...");
 x = 4.0;
 mystereo.set_volume(x * factor);
 System.out.println(mystereo.get_volume());
 }
}
```

---

# Method invocation: Memory structure

Before calling `mystereo.set_volume(x*factor)`



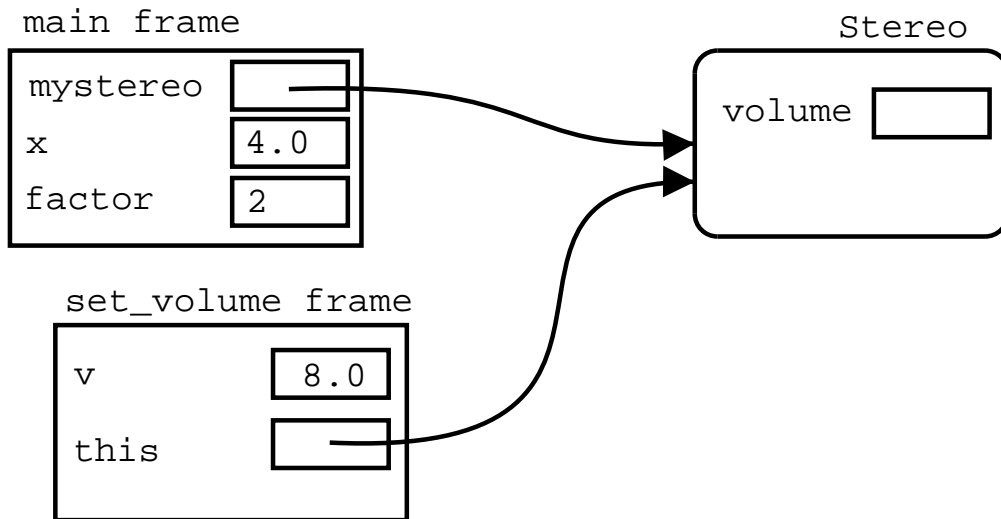
First its arguments ( `x*factor` ) are evaluated:

Evaluating `x*factor` in the main frame results in `8.0`

---

# Method invocation: Memory structure

A frame for `set_volume` is created, and the argument is assigned to the parameter: `v = 8.0;`

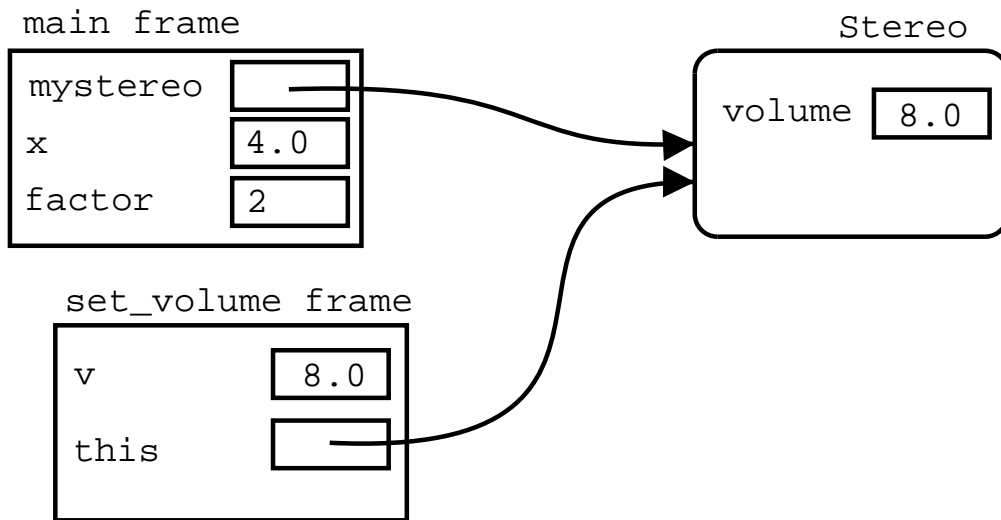




---

# Method invocation: Memory structure

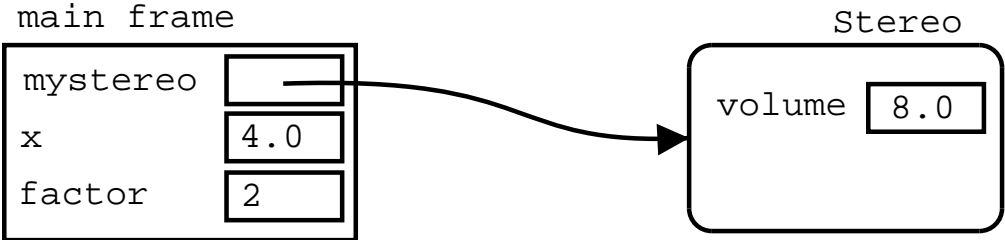
The current method (main) is suspended, and the body of the called method (set\_volume) is executed in the context of the current frame (the set\_volume frame):



---

# Method invocation: Memory structure

Finally the called method frame is discarded, and computation of the calling method (main) is resumed in the instruction immediately after the method call.



---

## Static methods

- Static methods have no direct effect on objects
- When calling a static method, the frame does not have a “this” reference
- Therefore they cannot access any object attributes
- They can access only:
  - Parameters
  - Local variables
  - static variables (shared between all objects in the class)
- Non-static methods can access both static and non-static methods and variables
- Static methods cannot access non-static methods and non-static instance variables

---

## Objects as first class values

- Objects can be passed as parameters and returned as values

```
public class Rabbit {
 void jump() { ... }
}
public class Cage {
 void shake(Rabbit a)
 {
 a.jump();
 }
 Rabbit create()
 {
 return new Rabbit();
 }
}
```

...elsewhere...

```
Rabbit bugs = new Rabbit();
Cage c = new Cage();
c.shake(bugs);
Rabbit wester = c.create();
```

---

## Objects as first class values

- Objects can be attributes of other objects

```
public class Rabbit {
 void jump() { ... }
}
public class Cage {
 Rabbit my_rabbit;
 void put(Rabbit a)
 {
 my_rabbit = a;
 }
 Rabbit get()
 {
 return my_rabbit;
 }
}
```

...elsewhere...

```
Rabbit bugs = new Rabbit();
Cage c = new Cage();
c.put(bugs);
Rabbit wester = c.get();
```

---

## Methods: divide and conquer

If a problem is too complex, then its solution should be done by dividing the problem into smaller subproblems, and solving each subproblem in a separate method. The solution to the entire problem is then achieved by combining the solutions of the subproblems (by calling the necessary methods.)

---

## Methods: *reusable* abstractions

- A method can be reused in different contexts
- Calling a method is “the same” as substituting its body in place of its call (replacing the parameters by the actual arguments,) but
- If we define a method, we can simply call it from more than one context without having to do copy and paste.

---

## Methods: reusable abstractions

Determining whether  $n$  is a prime number or not:

```
boolean result;
int i;

result = true;
i = 2;
while (i < n && result) {
 if (n % i == 0) {
 result = false;
 }
 i++;
}
```



---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static void print_primes(int m)
 {
 boolean result;
 int n;

 n = 1;
 while (n <= m) {

 // Find out if n is prime...
 if (result)
 System.out.println(n);
 n++;
 }
 }
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static void print_primes(int m)
 {
 boolean result;
 int i, n;

 n = 1;
 while (n <= m) {
 result = true;
 i = 2;
 while (i < n && result) {
 if (n % i == 0) {
 result = false;
 }
 i++;
 }
 if (result)
 System.out.println(n);
 n++;
 }
 }
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n)
 {
 boolean result;
 int i;

 result = true;
 i = 2;
 while (i < n && result) {
 if (n % i == 0) {
 result = false;
 }
 i++;
 }
 return result;
 }
 //... rest of the class
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static void print_primes(int m)
 {
 boolean result;
 int i, n;

 n = 1;
 while (n <= m) {
 result = true;
 i = 2;
 while (i < n && result) {
 if (n % i == 0) {
 result = false;
 }
 i++;
 }
 if (result)
 System.out.println(n);
 n++;
 }
 }
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n) { ... }

 static void print_primes(int m)
 {
 boolean result;
 int n;

 n = 1;
 while (n <= m) {
 result = is_prime(n);
 if (result)
 System.out.println(n);
 n++;
 }
 }
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n) { ... }

 static void print_primes(int m)
 {
 int n;

 n = 1;
 while (n <= m) {
 if (is_prime(n))
 System.out.println(n);
 n++;
 }
 }
}
```

---

## Methods: reusable abstractions

Problem: given three numbers, determine whether all of them are prime or their sum is prime

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n) { ... }

 static void threenumbers(int a, int b, int c)
 {
 if (is_prime(a) && is_prime(b) && is_prime(c)
 || is_prime(a+b+c)) {
 return true;
 }
 return false;
 }
}
```



---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n) { ... }

 static void threenumbers(int a, int b, int c)
 {
 return (is_prime(a) && is_prime(b) && is_prime
 || is_prime(a+b+c));
 }
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n) { ... }

 static void threenumbers(int a, int b, int c)
 {
 boolean result1, result2, result3, result4;
 int i;

 result1 = true;
 i = 2;
 while (i < a && result1) {
 if (a % i == 0) {
 result1 = false;
 }
 i++;
 }
 result2 = true;
 i = 2;
 while (i < b && result2) {
 if (b % i == 0) {
 result2 = false;
 }
 }
 }
}
```

---

```
 }
 i++;
}
result3 = true;
i = 2;
while (i < c && result3) {
 if (c % i == 0) {
 result3 = false;
 }
 i++;
}
result4 = true;
i = 2;
while (i < a+b+c && result4) {
 if ((a+b+c) % i == 0) {
 result4 = false;
 }
 i++;
}
return result1 && result2 && result3 || result4;
}
}
```

---

## Methods: reusable abstractions

```
public class MyMathProcedures {
 static boolean is_prime(int n)
 {
 boolean result;
 int i;

 result = true;
 i = 2;
 while (i < Math.sqrt(n) && result) {
 if (n % i == 0) {
 result = false;
 }
 i++;
 }
 return result;
 }
 //... rest of the class
}
```

---

# Recursion

- A recursive method is a method that calls itself (directly or indirectly.)
- A recursive definition is a definition of something in terms of itself
- Some recursive definitions don't make sense, (e.g. from Webster's: growl: to utter a growl), but others do
- For example:
  - A *list of numbers* is either:
    - \* A single number, or
    - \* A number followed by a list of numbers.
  - For example:
    - \* 5 is a list of numbers
    - \* 7, 5 is a list of numbers (because 5 is a list)
    - \* 6, 7, 5 is a list of numbers (because 7, 5 is a list)
    - \* 8, 6, 7, 5 is a list of numbers (because 6, 7, 5 is a list)

---

## Recursive functions

- Factorial: the factorial of a natural number  $n$ , written  $n!$  is the multiplication of the first  $n$  positive integers, i.e.

$$n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n - 2) \cdot (n - 1) \cdot n \quad (1)$$

But note that

$$1 \cdot 2 \cdot 3 \cdot \dots \cdot (n - 2) \cdot (n - 1) = (n - 1)! \quad (2)$$

So by (1) and (2) we get

$$n! = (n - 1)! \cdot n \quad (3)$$

But we have to assume a “base case”, by defining

$$0! = 1 \quad (4)$$

---

## Recursive functions (contd.)

Hence, (3) and (4) together gives us an alternative, and recursive definition of (1):

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n - 1)! \cdot n & \text{otherwise} \end{cases}$$

This can be implemented as a static recursive method:

```
static int factorial(int n)
{
 if (n == 0) {
 return 1;
 }
 return factorial(n-1)*n;
}
```

---

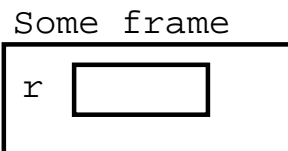
## Execution of recursive methods

Consider the following client for this factorial function:

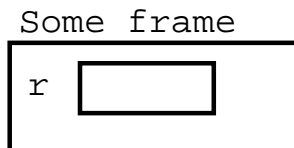
```
int r;
r = factorial(4);
```

Its execution proceeds as follows:

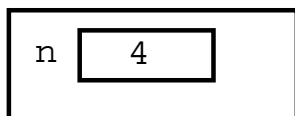
This is executed in some frame:



When we call `factorial(4)`; a new frame for the method is created:



factorial frame



We execute the body of factorial; `n` is not 0 so we execute

```
return factorial(n-1)*n;
```

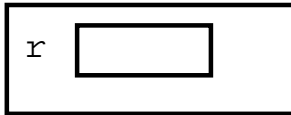
which in this frame is the same as

```
return factorial(4-1)*4;
```

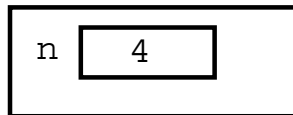


---

Some frame



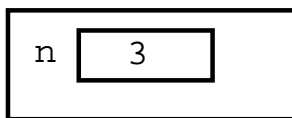
factorial frame



pending computation:

`return factorial(3)*4;`

factorial frame



Again, we execute the body of factorial;  
again, n is not 0 so we execute

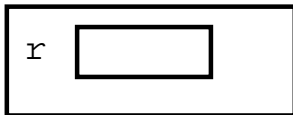
`return factorial(n-1)*n;`

which in this frame is the same as

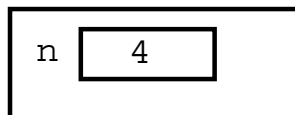
`return factorial(3-1)*3;`

---

Some frame



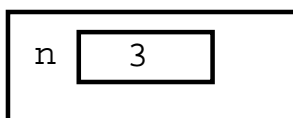
factorial frame



pending computation:

`return factorial(3)*4;`

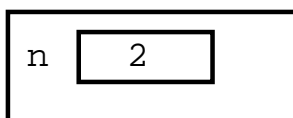
factorial frame



pending computation:

`return factorial(2)*3;`

factorial frame



Again, we execute the body of factorial;  
again, n is not 0 so we execute

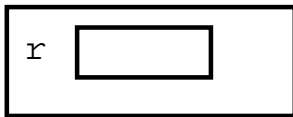
`return factorial(n-1)*n;`

which in this frame is the same as

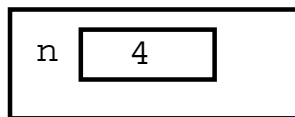
`return factorial(2-1)*2;`

---

Some frame



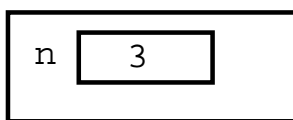
factorial frame



pending computation:

`return factorial(3)*4;`

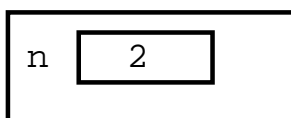
factorial frame



pending computation:

`return factorial(2)*3;`

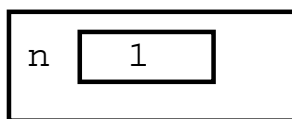
factorial frame



pending computation:

`return factorial(1)*2;`

factorial frame



Again, we execute the body of factorial;  
again, n is not 0 so we execute

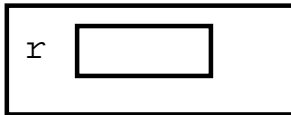
`return factorial(n-1)*n;`

which in this frame is the same as

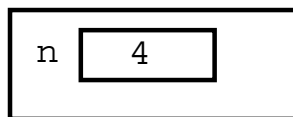
`return factorial(1-1)*1;`

---

Some frame



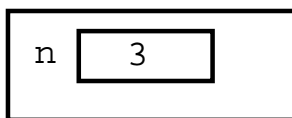
factorial frame



pending computation:

`return factorial(3)*4;`

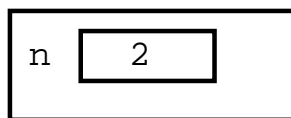
factorial frame



pending computation:

`return factorial(2)*3;`

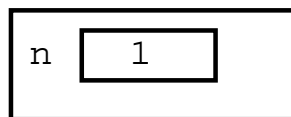
factorial frame



pending computation:

`return factorial(1)*2;`

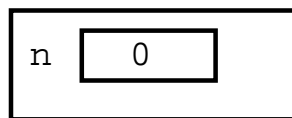
factorial frame



pending computation:

`return factorial(0)*1;`

factorial frame



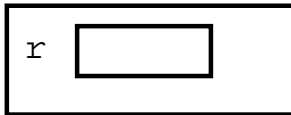
Now, we have reached the base case, and n is 0, so we execute:

`return 1;`

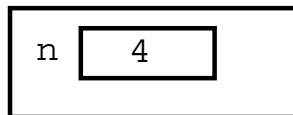
We get rid of the frame, and pass the returned value to the caller

---

Some frame



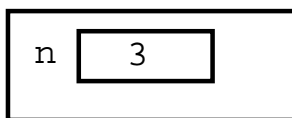
factorial frame



pending computation:

```
return factorial(3)*4;
```

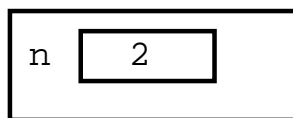
factorial frame



pending computation:

```
return factorial(2)*3;
```

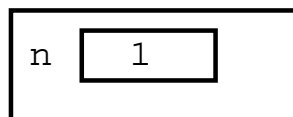
factorial frame



pending computation:

```
return factorial(1)*2;
```

factorial frame



The pending computation here was:

```
return factorial(0)*1;
```

and the method called `factorial(0)`

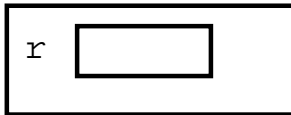
returned 1, so this pending computation is now:

```
return 1*1;
```

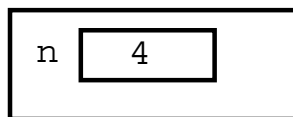
We get rid of the frame, and pass the returned value to the caller

---

Some frame



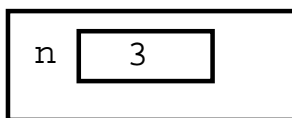
factorial frame



pending computation:

`return factorial(3)*4;`

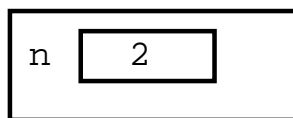
factorial frame



pending computation:

`return factorial(2)*3;`

factorial frame



The pending computation here was:

`return factorial(1)*2;`

and the method called `factorial(1)`

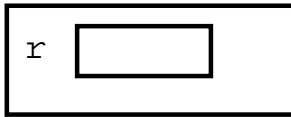
returned 1, so this pending computation is now:

`return 1*2;`

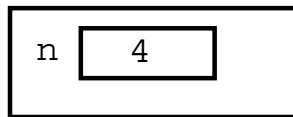
We get rid of the frame, and pass the returned value to the caller

---

Some frame



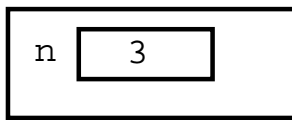
factorial frame



pending computation:

```
return factorial(3)*4;
```

factorial frame



The pending computation here was:

```
return factorial(2)*3;
```

and the method called `factorial(2)`

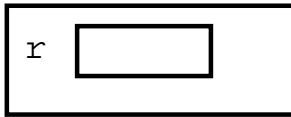
returned 2, so this pending computation is now:

```
return 2*3;
```

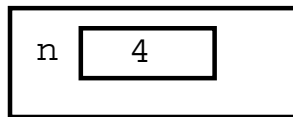
We get rid of the frame, and pass the returned value to the caller

---

Some frame



factorial frame



The pending computation here was:

```
return factorial(3)*4;
```

and the method called `factorial(3)`

returned 6, so this pending computation is now:

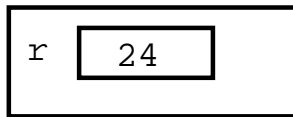
```
return 6*4;
```

We get rid of the frame, and pass the returned value to the caller



---

Some frame



The pending computation here was:

```
r = factorial(4);
```

which returned 24, so this pending computation is now:

```
r = 24;
```

---

## Recursion on other types

- Problem: given a string  $s$ , return the reverse of the string
- Analysis:
  - Notation:
    - \*  $\text{rev}(s)$  is the reverse of  $s$
    - \*  $s_i$  is the  $i$ -th character of  $s$
    - \*  $\text{len}(s)$  is the length of  $s$
    - \*  $\text{rest}(s)$  is the string  $s$  without its first character  $s_0$   
(i.e.  $\text{rest}(s) = s_1s_2\dots s_n$  where  $n = \text{len}(s) - 1$ )
  - Formal definition of reverse:

$$\text{rev}(s) = \begin{cases} "" & \text{if } s = "" \\ \text{rev}(\text{rest}(s)) + s_0 & \text{otherwise} \end{cases}$$

---

## Reverse (contd.)

- For example:

$$\begin{aligned}\text{rev}(\text{"abcd"}) &= \text{rev}(\text{"bcd"}) + 'a' \\ &= (\text{rev}(\text{"cd"}) + 'b') + 'a' \\ &= ((\text{rev}(\text{"d"}) + 'c') + 'b') + 'a' \\ &= (((\text{rev}(\text{""}) + 'd') + 'c') + 'b') + 'a' \\ &= (((\text{""} + 'd') + 'c') + 'b') + 'a' \\ &= ((\text{"d"} + 'c') + 'b') + 'a' \\ &= (\text{"dc"} + 'b') + 'a' \\ &= \text{"dcb"} + 'a' \\ &= \text{"dcba"}\end{aligned}$$

---

## Reverse (contd.)

```
public class MoreStringOperations {
 static String reverse(String s)
 {
 if (s.equals("")) {
 return "";
 }
 return reverse(rest(s))+s.charAt(0);
 }
 static String rest(String s)
 {
 String result = "";
 int i = 1;
 while (i < s.length()) {
 result = result + s.charAt(i);
 i++;
 }
 return result;
 }
}
```

---

## Double recursion

- Problem: Compute the  $n$ -th Fibonacci number
- Analysis: The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, ... is defined by:

$$fib(n) = \begin{cases} 1 & \text{if } n \leq 2 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}$$

- Implementation:

```
static int fib(int n)
{
 if (n <= 2) {
 return 1;
 }
 return fib(n-1)+fib(n-2);
}
```

---

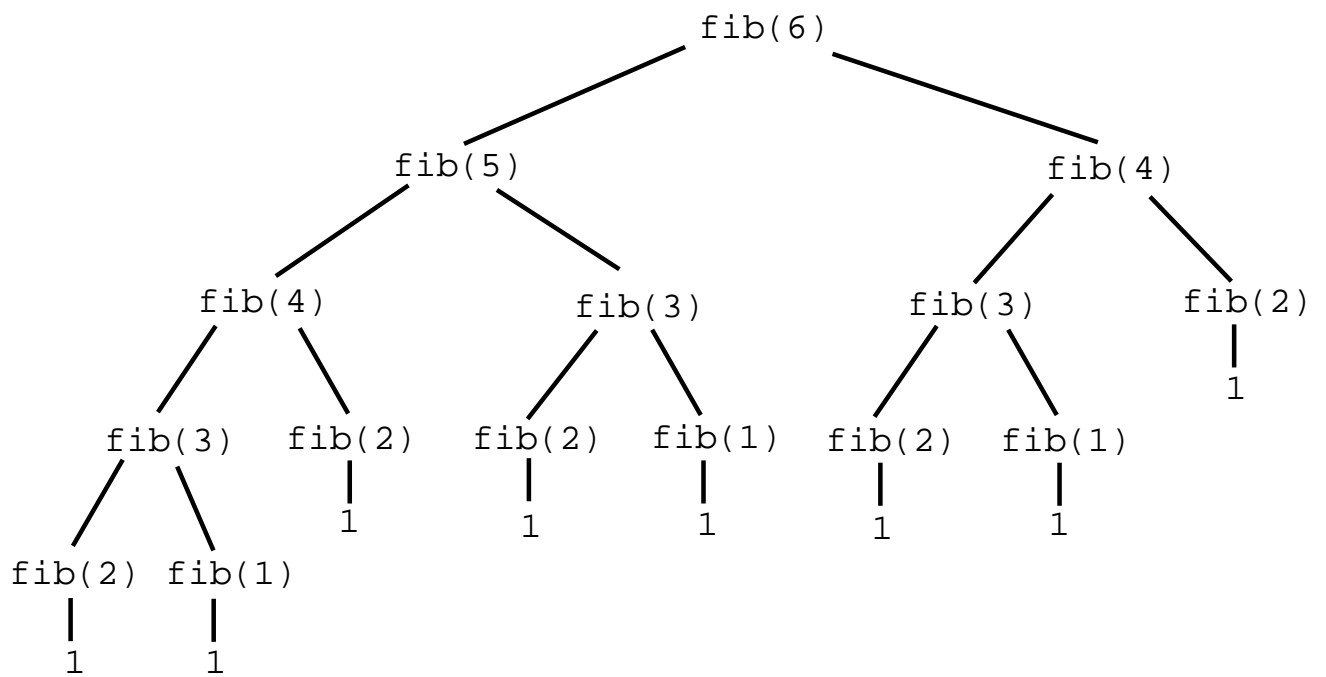
## Iteration vs recursion

- Iterative solution to the Fibonacci problem:

```
static int fib(int n)
{
 int a, b, c, i;
 a = 1;
 b = 1;
 c = 1;
 i = 3;
 while (i <= n) {
 c = a + b;
 a = b;
 b = c;
 i++;
 }
 return c;
}
```

---

# Execution trees



---

## Method overloading

- There can be several (static or not) methods with the same name...
- ...but the type or number of parameters must be different



---

## Example

```
public class A {
 void f(int x)
 {
 System.out.println("one: "+x)
 }
 void f(boolean x)
 {
 System.out.println("two: "+x)
 }
}
public class B {
 void g()
 {
 A u = new A();
 u.f(5);
 u.f(false);
 }
}
```

---

## Same for static methods

```
public class A {
 static void f(int x)
 {
 System.out.println("one: "+x)
 }
 static void f(boolean x)
 {
 System.out.println("two: "+x)
 }
}
public class B {
 void g()
 {
 A.f(5);
 A.f(false);
 }
}
```

---

The end