
Announcements

- Final exam: April 20th at 9:00am, at the GYM
- Review tutorial: April 12th from 4:00pm to 6:00pm at ENGMC 13
- Course evaluations on Minerva (before April 13th:)
 - Login to **Minerva for students** (from <http://www.mcgill.ca>)
 - Select **Student Menu** (or a pop-up window will appear)
 - Click on **MOLE** - McGill Online Evaluations
 - Select COMP-202
 - Fill out the evaluation (it's anonymous.)

Recursive data-structures

- For example:
 - A *list of data* is either:
 - * An *empty list* [], or
 - * A *pair* consisting of:
 - Some data, and
 - A list of data.
 - For example:
 - * [] is a list
 - * [5, []] is a list
 - * [7, [5, []]] is a list
 - * [6, [7, [5, []]]] is a list
 - * [8, [6, [7, [5, []]]]] is a list

Recursive data-structures

EmptyList

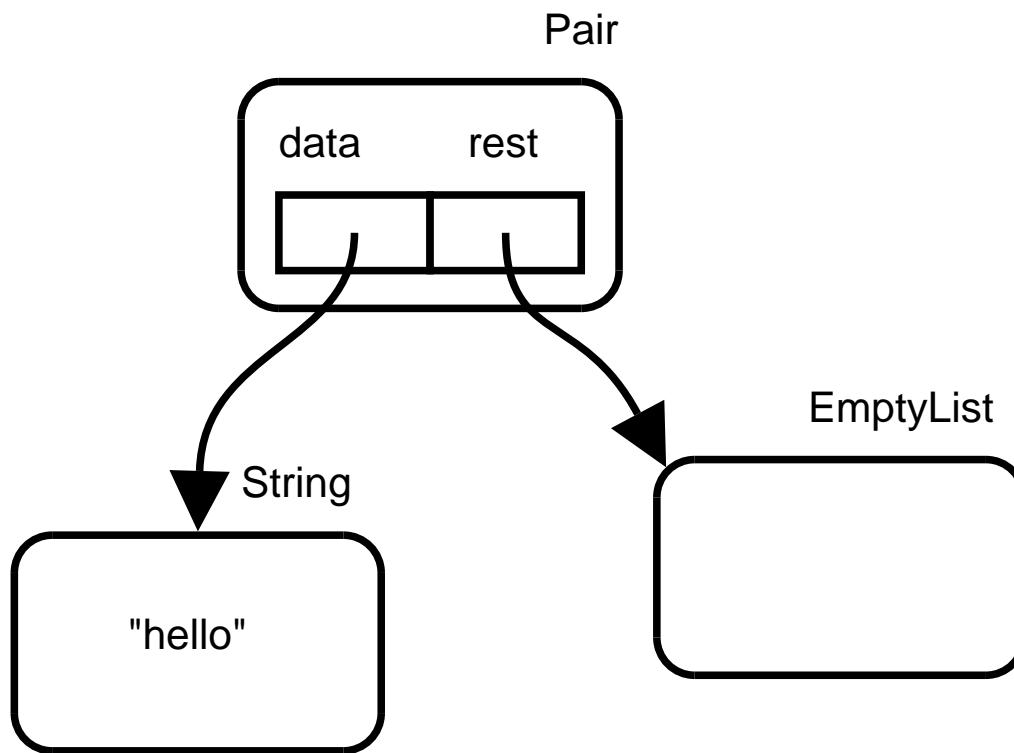


Recursive data-structures

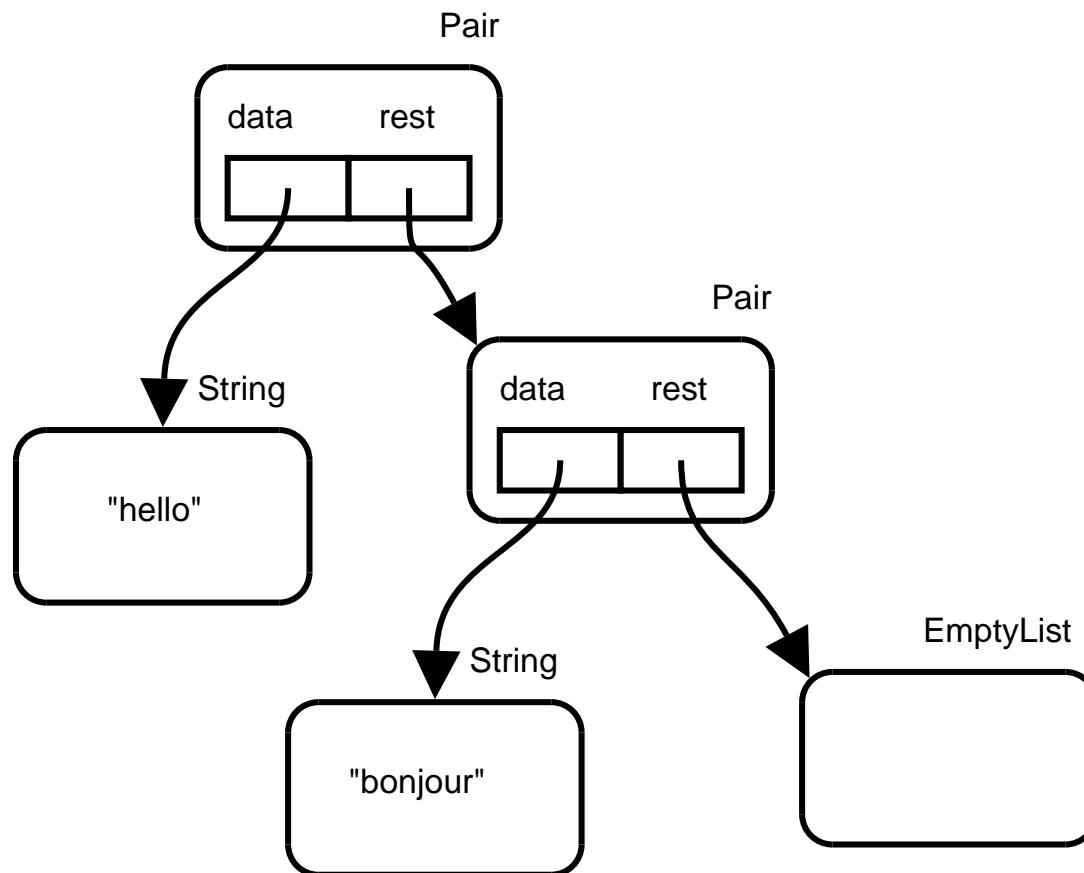
Pair



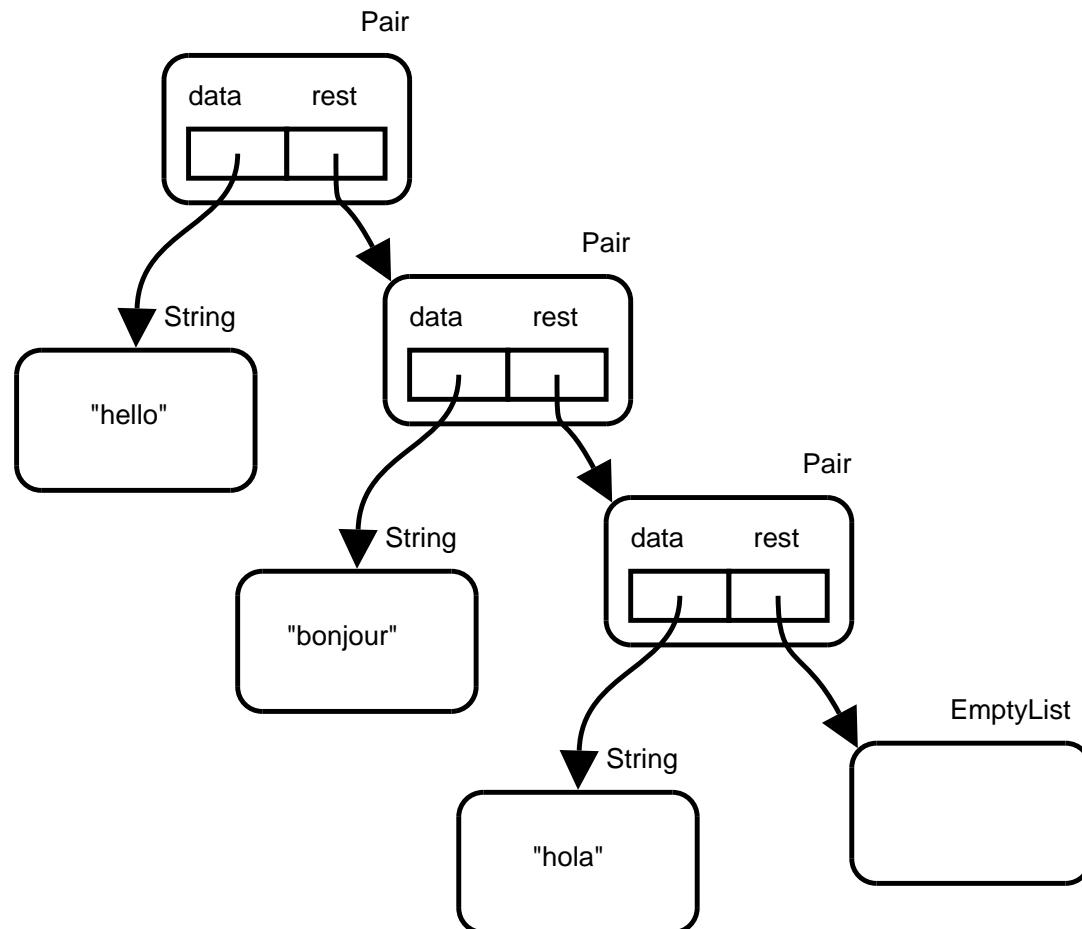
Recursive data-structures



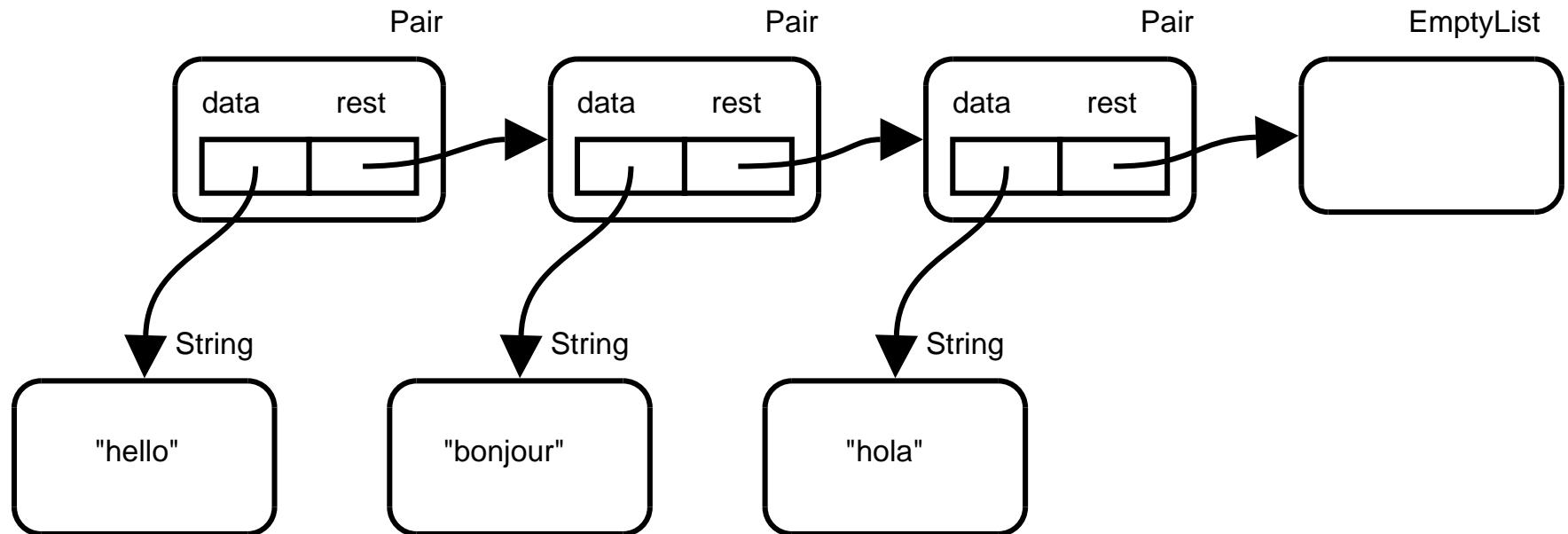
Recursive data-structures



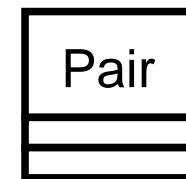
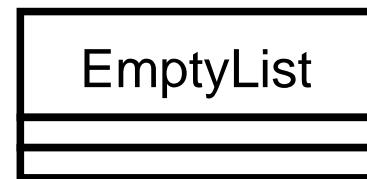
Recursive data-structures



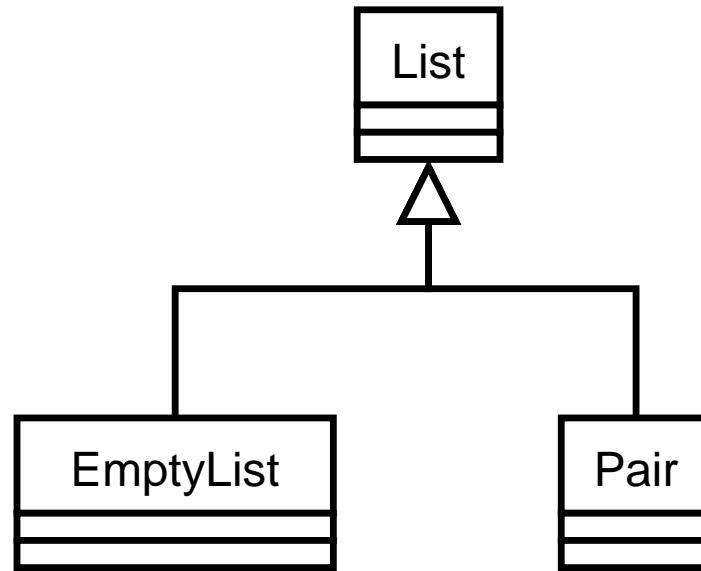
Recursive data-structures



Recursive data-structures

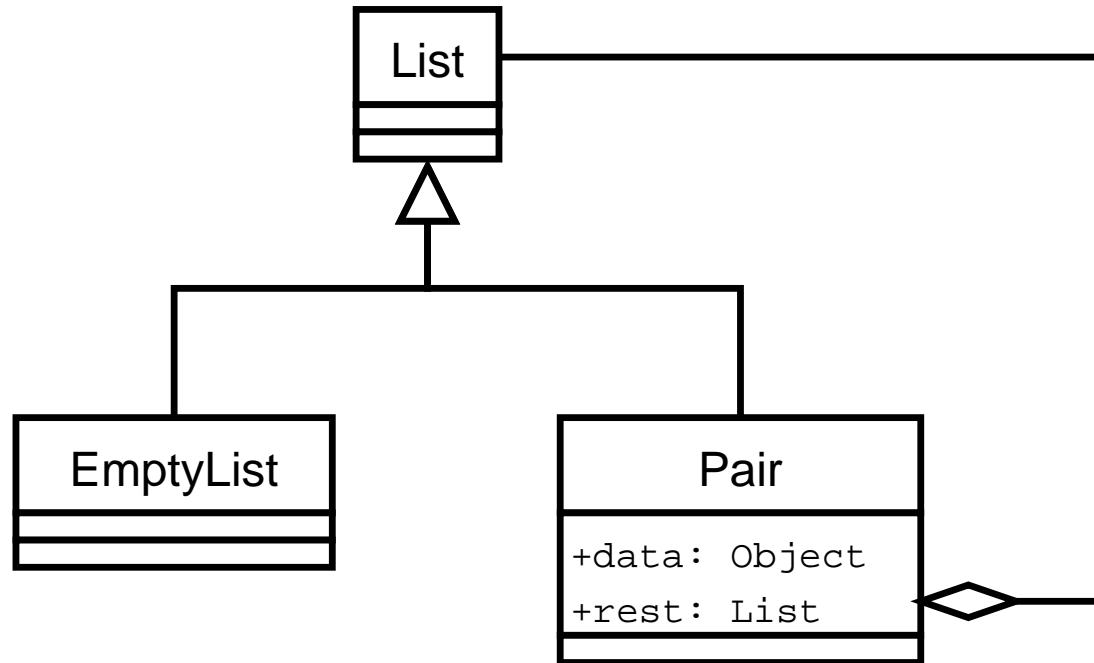


Recursive data-structures



- An empty list *is a* list
 - A pair *is a* list
-

Recursive data-structures



- A pair *has a* data object
 - A pair *has a* list (a reference to the rest or the list)
-

Recursive data-structures

```
class List
{
}

class EmptyList extends List
{
}

class Pair extends List
{
    Object data;
    List   rest;
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

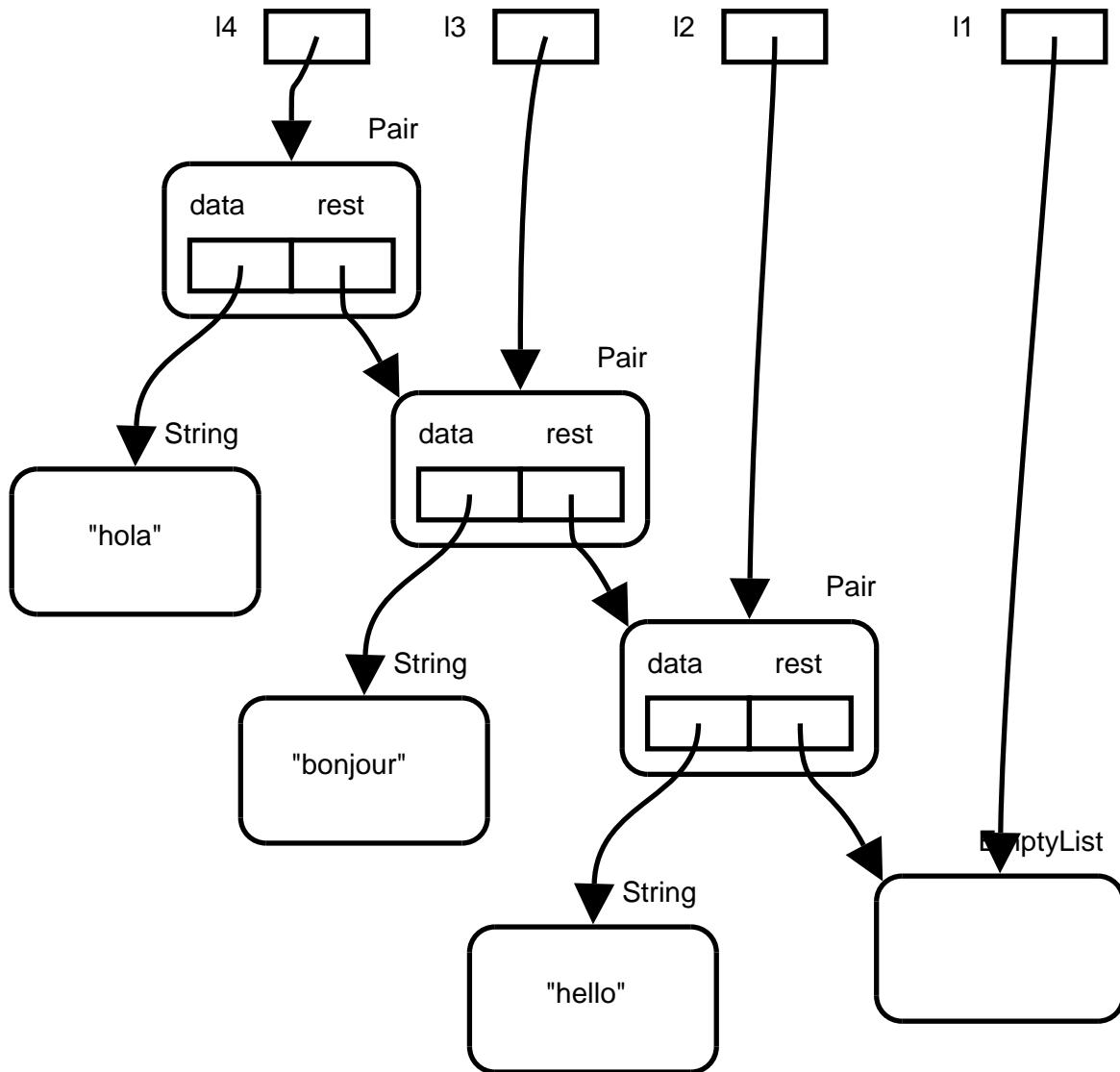
    Pair(Object d, List l)
    {
        data = d;
        rest = l;
    }

    Object getData() { return data; }
    List  getRest() { return rest; }
}
```

Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l1 = new EmptyList();
        List l2 = new Pair("hello", l1);
        List l3 = new Pair("bonjour", l2);
        List l4 = new Pair("hola", l3);
    }
}
```

Recursive data-structures

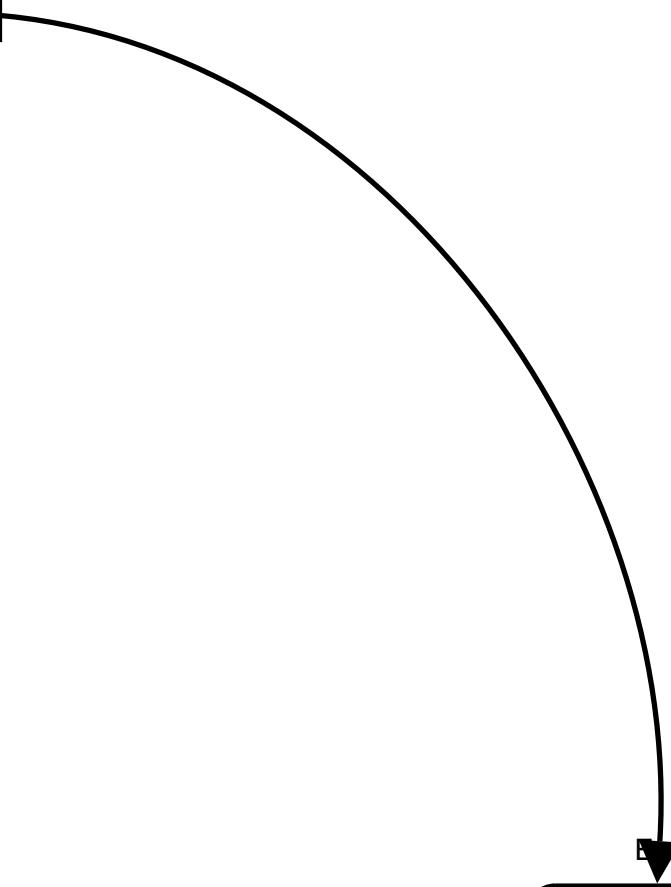


Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l = enter_list(4);
    }
    static List enter_list(int n)
    {
        Scanner scanner = new Scanner(System.in);
        List mylist = new EmptyList();
        int i = 1;
        while (i <= n)
        {
            System.out.print("Enter a word: ");
            String word = scanner.nextLine();
            mylist = new Pair(word, mylist);
            i++;
        }
        return mylist;
    }
}
```

Recursive data-structures

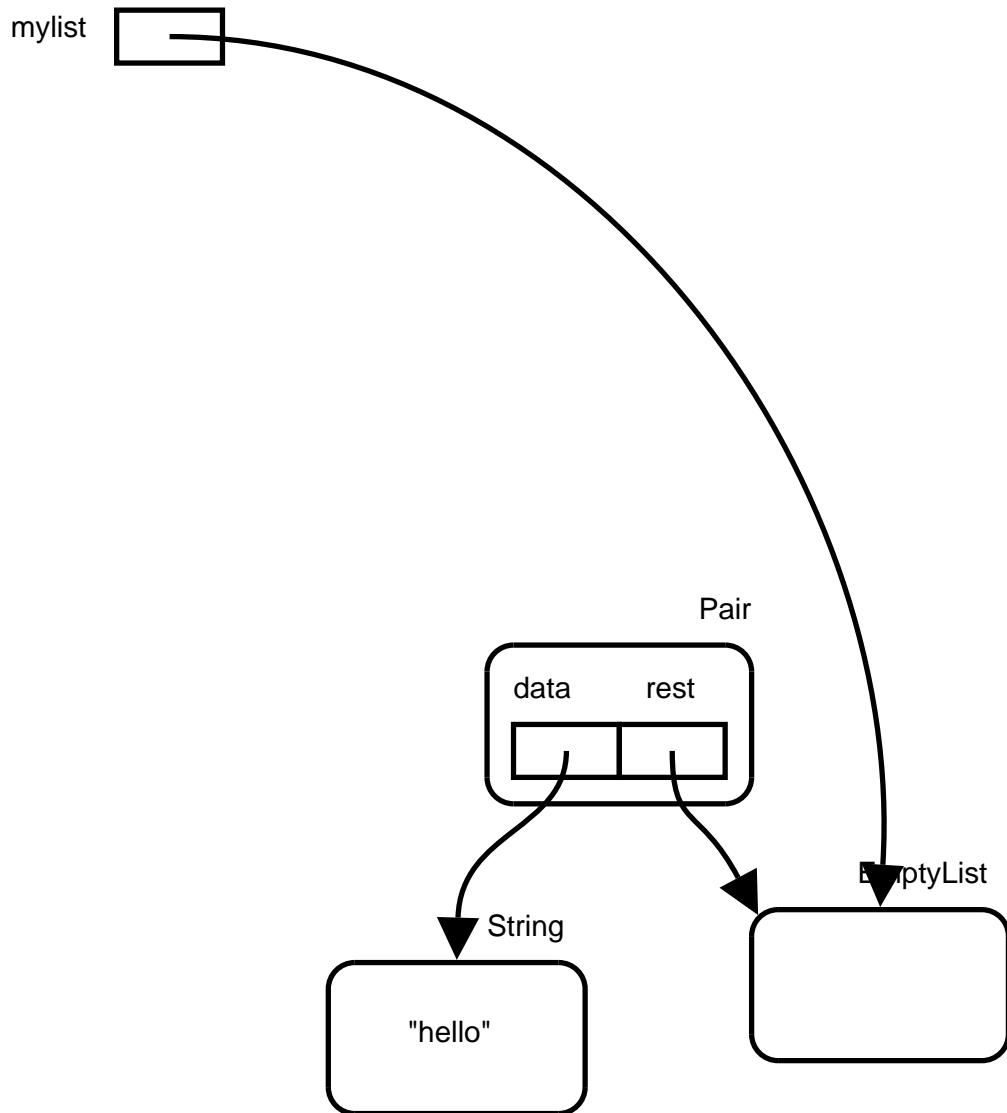
mylist



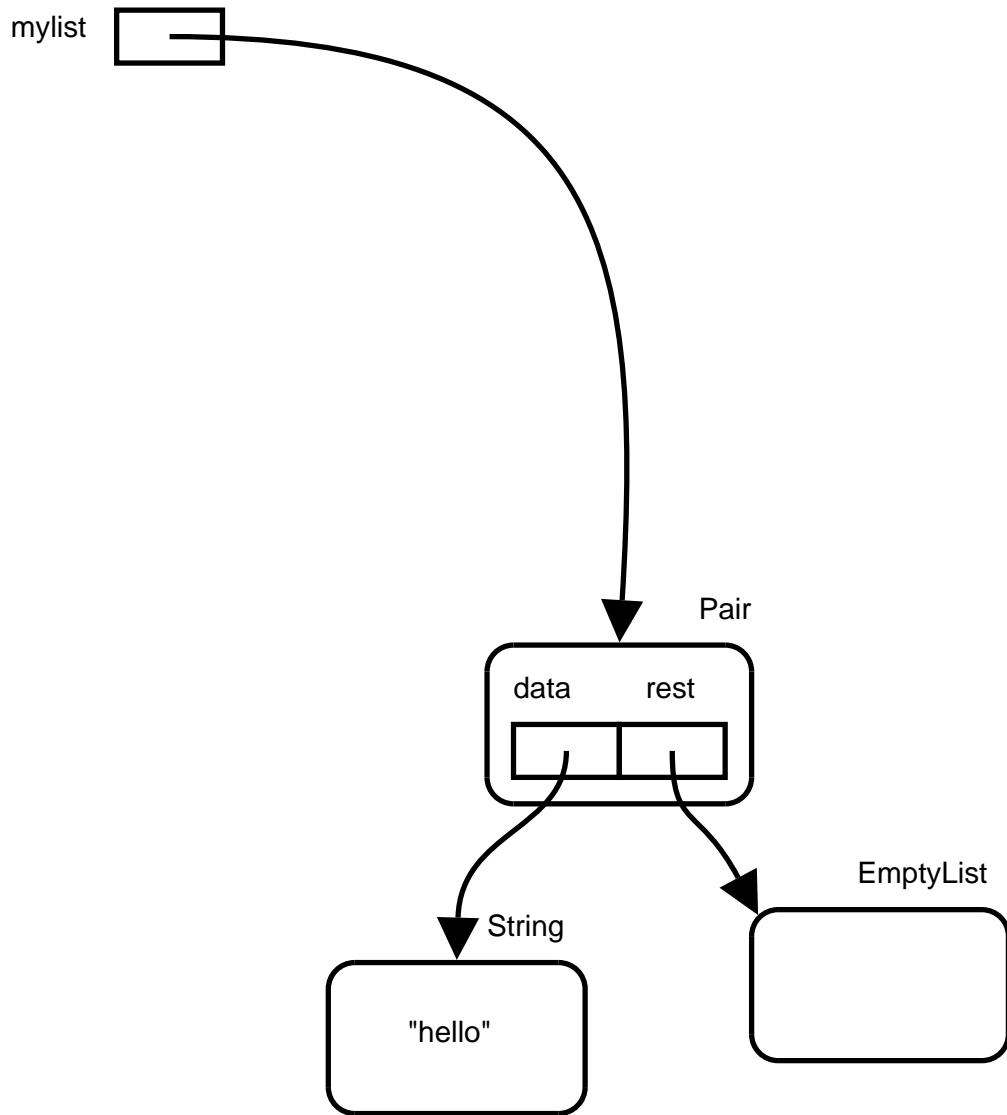
EmptyList



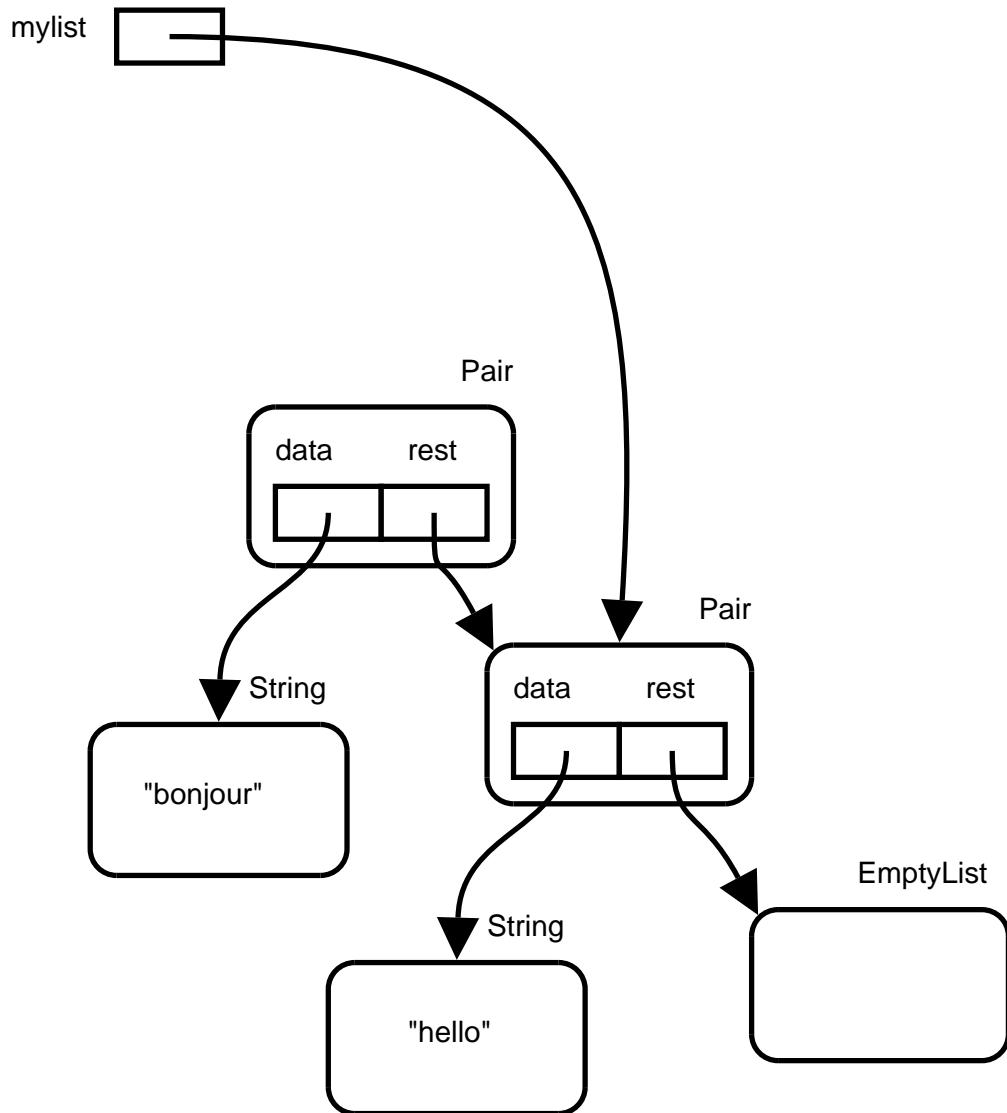
Recursive data-structures



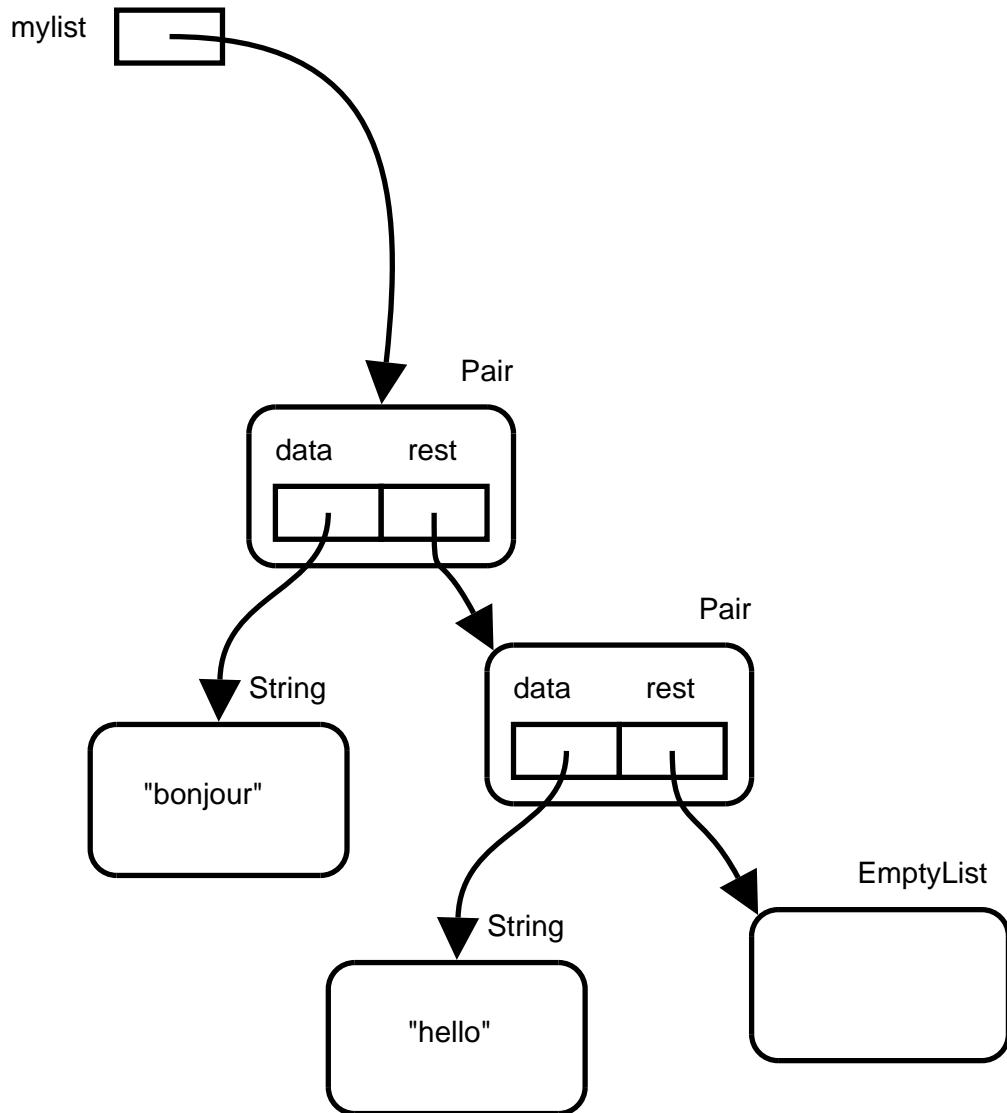
Recursive data-structures



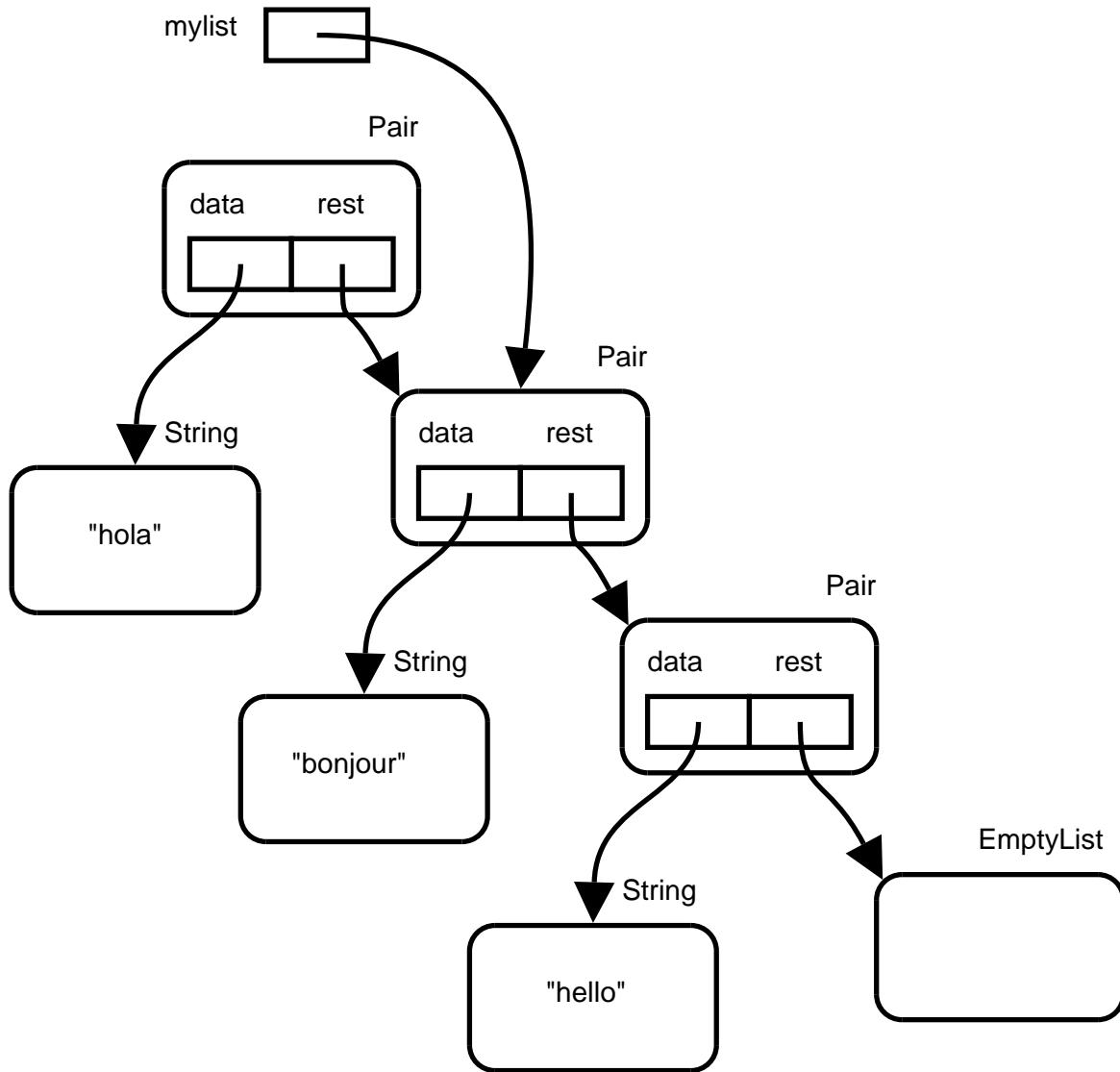
Recursive data-structures



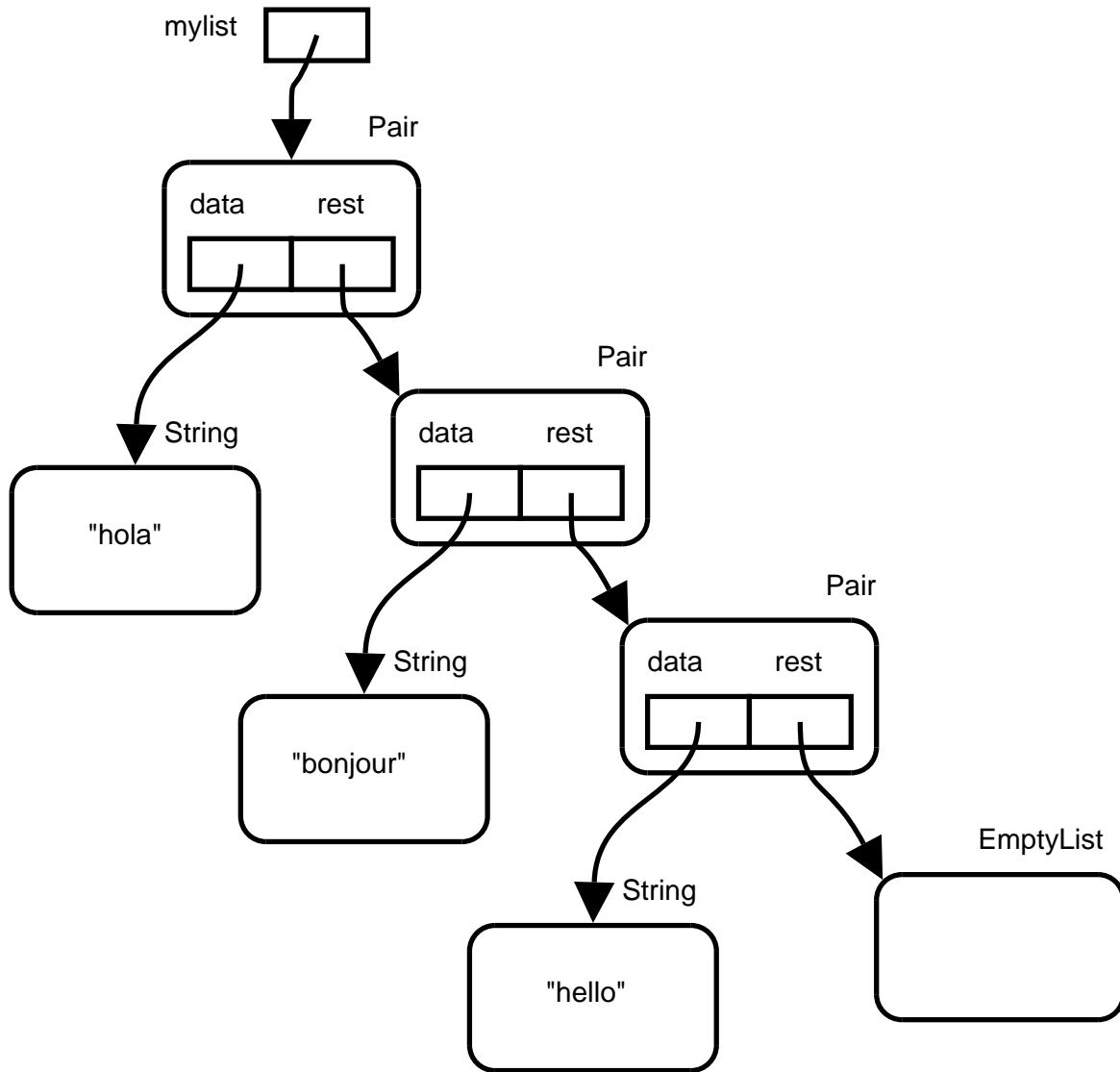
Recursive data-structures



Recursive data-structures



Recursive data-structures



Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l = enter_list(4);
        print_list(l);
    }

    static List enter_list()
    { ... }

    static void print_list(List l)
    { ... }
}
```

Recursive data-structures

To print a list l :

1. If l is an empty list:
 - (a) print $''$
2. Otherwise (it is a pair)
 - (a) print the *data* of the pair, and
 - (b) print the *rest* of the list

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    Pair(Object d, List l)
    {
        data = d;
        rest = l;
    }

    Object getData() { return data; }
    List  getRest() { return rest; }
}
```

Recursive data-structures

```
public class ListTest
{
    ...
    static void print_list(List l)
    {
        if (l instanceof EmptyList)
        {
            System.out.print("");
        }
        else
        {
            Pair p = (Pair)l;
            Object data = p.getData();
            List rest = p.getRest();
            System.out.print(data + ", ");
            print_list(rest);
        }
    }
}
```

Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l = enter_list(4);
        print_list(l);
    }

    static List enter_list()
    { ... }

    static void print_list(List l)
    { ... }

    static boolean member(Object item, List l)
    { ... }
}
```

Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l = enter_list(4);
        print_list(l);
        if (member("beer", l))
        {
            System.out.print("It's there");
        }
    }

    static List enter_list()
    { ... }

    static void print_list(List l)
    { ... }

    static boolean member(Object item, List l)
    { ... }
}
```

Recursive data-structures

To determine whether an item x is in a list $/$, do:

1. If $/$ is an empty list:
 - (a) return false
2. Otherwise (it is a pair)
 - (a) If the *data* of the pair is equal to x :
 - i. return true
 - (b) Otherwise:
 - i. determine whether x is in the *rest* of the list, and return the result of that

Recursive data-structures

```
static boolean member(Object item, List l)
{
    if (l instanceof EmptyList)
    {
        return false;
    }
    else
    {
        Pair p = (Pair)l;
        Object data = p.getData();
        if (data.equals(item))
        {
            return true;
        }
        else
        {
            List rest = p.getRest();
            return member(item, rest);
        }
    }
}
```

Recursive data-structures

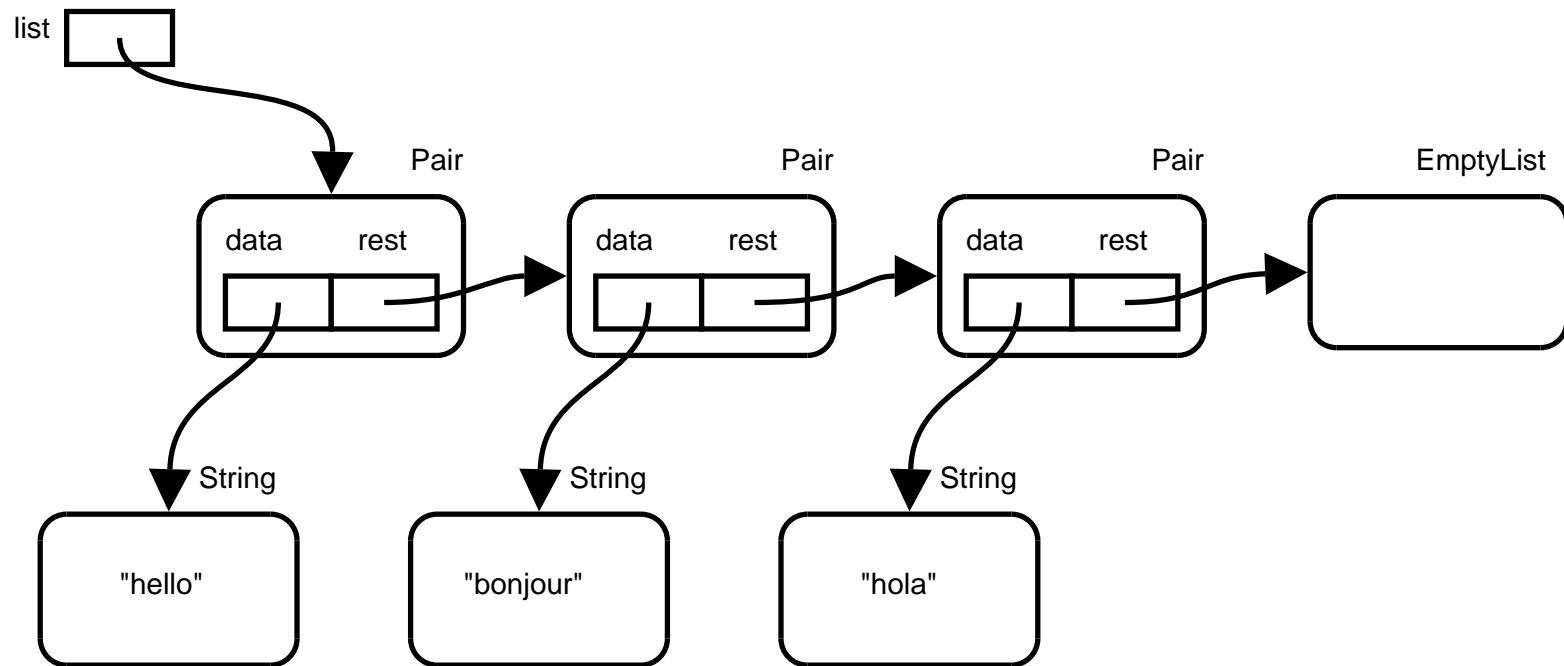
To obtain item number n in a list l , do:

1. If l is an empty list:
 - (a) throw an exception
2. Otherwise (it is a pair)
 - (a) If n is 0:
 - i. return the *data* of the pair
 - (b) Otherwise:
 - i. return item number $n - 1$ of the *rest* of the list

Recursive data-structures

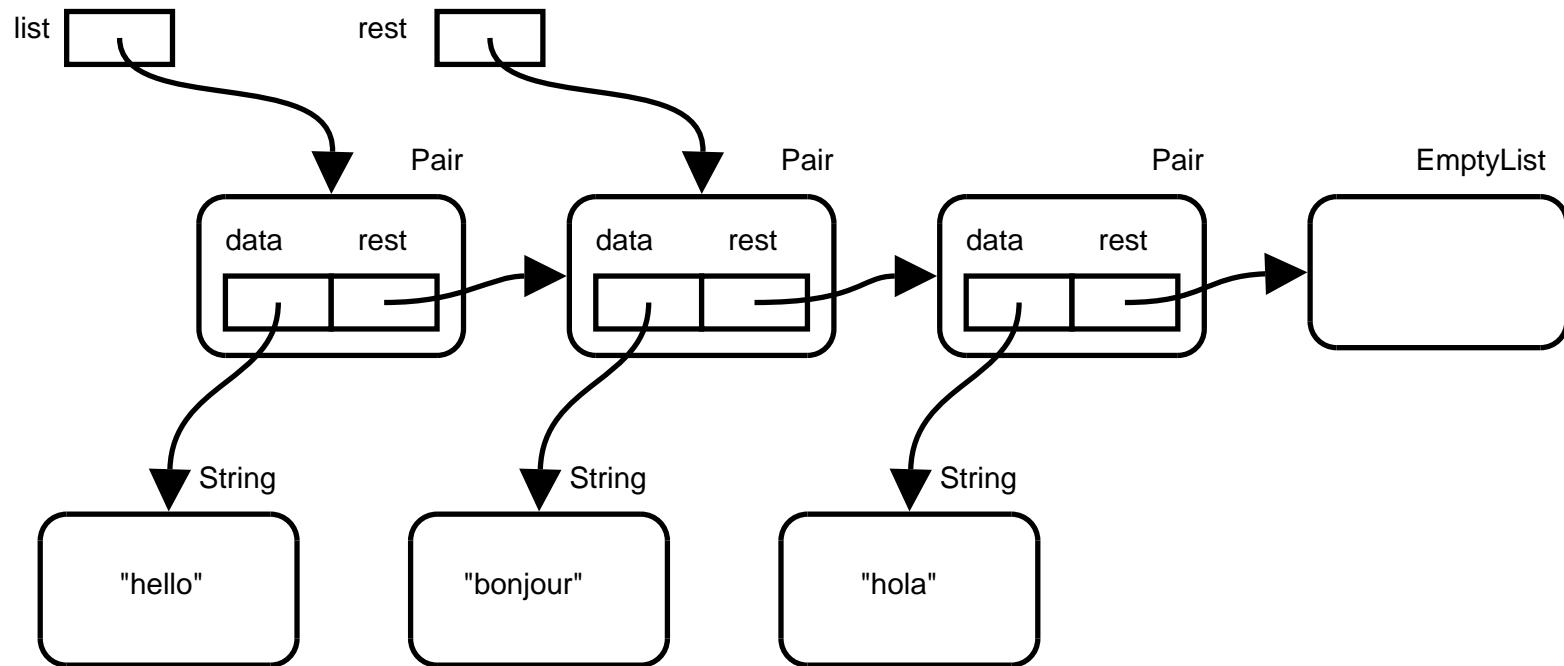
```
static Object get(List l, int n)
throws NoSuchElementException
{
    if (l instanceof EmptyList)
    {
        throw new NoSuchElementException();
    }
    else
    {
        Pair p = (Pair)l;
        if (n == 0)
        {
            Object data = p.getData();
            return data;
        }
        else
        {
            List rest = p.getRest();
            return get(rest, n - 1);
        }
    }
}
```

Recursive data-structures



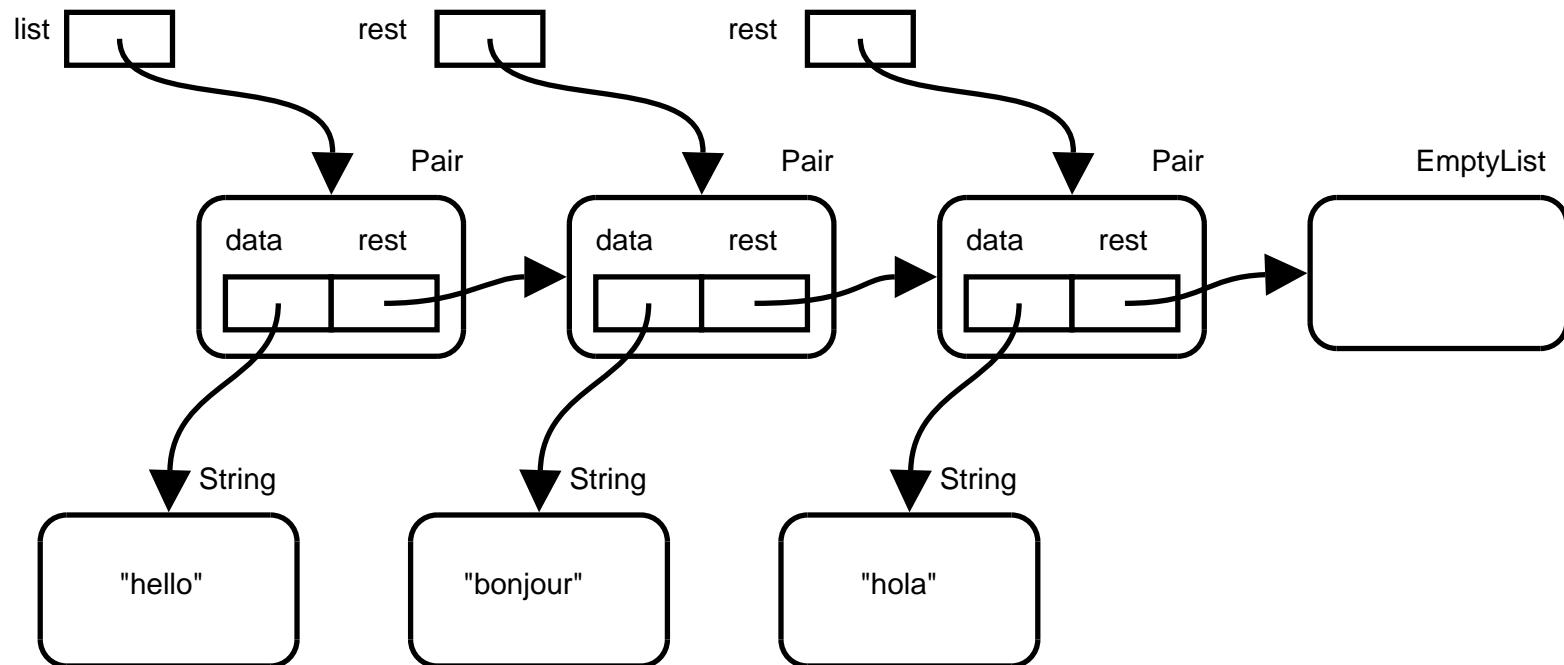
`get(list, 2)`

Recursive data-structures



```
return get(rest, 1)
```

Recursive data-structures



```
return get(rest, 0)
```

returns "hola"

Recursive data-structures

Design principle:

Operations on a particular kind of object should be implemented as methods in the object's class

Recursive data-structures

```
class List
{
    static void print_list(List l)
    { ... }
    static boolean member(Object item, List l)
    { ... }
    static Object get(List l, int n)
    { ... }
}
```

Recursive data-structures

```
class List
{
    void print_list()
    { ... }
    boolean member(Object item)
    { ... }
    Object get(int n)
    { ... }
}
```

Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l = enter_list(4);
        print_list(l);
        if (member("beer", l))
        {
            System.out.print("It's there");
        }
    }

    static List enter_list()
    { ... }

    static void print_list(List l)
    { ... }

    static boolean member(Object item, List l)
    { ... }
}
```

Recursive data-structures

```
public class ListTest
{
    public static void main(String[] args)
    {
        List l = enter_list(4);
        l.print_list();
        if (l.member("beer"))
        {
            System.out.print("It's there");
        }
    }
    static List enter_list()
    { ... }
}
```

Recursive data-structures

```
class ListTest
{
    static void print_list(List l)
    {
        if (l instanceof EmptyList)
        {
            System.out.print("");
        }
        else
        {
            Pair p = (Pair)l;
            Object data = p.getData();
            List rest = p.getRest();
            System.out.print(data + ", ");
            print_list(rest);
        }
    }
}
```

Recursive data-structures

```
class List
{
    void print_list()
    {
        if (this instanceof EmptyList)
        {
            System.out.print("");
        }
        else
        {
            Pair p = (Pair)this;
            Object data = p.getData();
            List rest = p.getRest();
            System.out.print(data + ", ");
            rest.print_list();
        }
    }
}
```

Recursive data-structures

```
static boolean member(Object item, List l)
{
    if (l instanceof EmptyList)
    {
        return false;
    }
    else
    {
        Pair p = (Pair)l;
        Object data = p.getData();
        if (data.equals(item))
        {
            return true;
        }
        else
        {
            List rest = p.getRest();
            return member(item, rest);
        }
    }
}
```

Recursive data-structures

```
class List {  
    boolean member(Object item)  
{  
        if (this instanceof EmptyList)  
        {  
            return false;  
        }  
        else  
        {  
            Pair p = (Pair)this;  
            Object data = p.getData();  
            if (data.equals(item))  
            {  
                return true;  
            }  
            else  
            {  
                List rest = p.getRest();  
                return rest.member(item);  
            }  
        }  
    }  
}
```

Recursive data-structures

```
static Object get(List l, int n)
throws NoSuchElementException
{
    if (l instanceof EmptyList)
    {
        throw new NoSuchElementException();
    }
    else
    {
        Pair p = (Pair)l;
        if (n == 0)
        {
            Object data = p.getData();
            return data;
        }
        else
        {
            List rest = p.getRest();
            return get(rest, n - 1);
        }
    }
}
```

Recursive data-structures

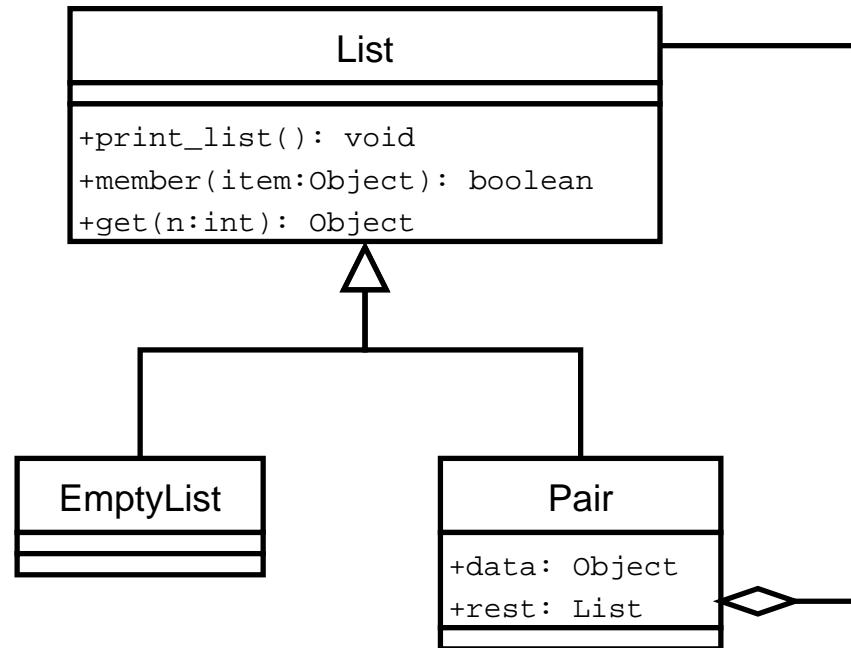
```
Object get(int n)
throws NoSuchElementException
{
    if (this instanceof EmptyList)
    {
        throw new NoSuchElementException();
    }
    else
    {
        Pair p = (Pair)this;
        if (n == 0)
        {
            Object data = p.getData();
            return data;
        }
        else
        {
            List rest = p.getRest();
            return rest.get(n - 1);
        }
    }
}
```

Recursive data-structures

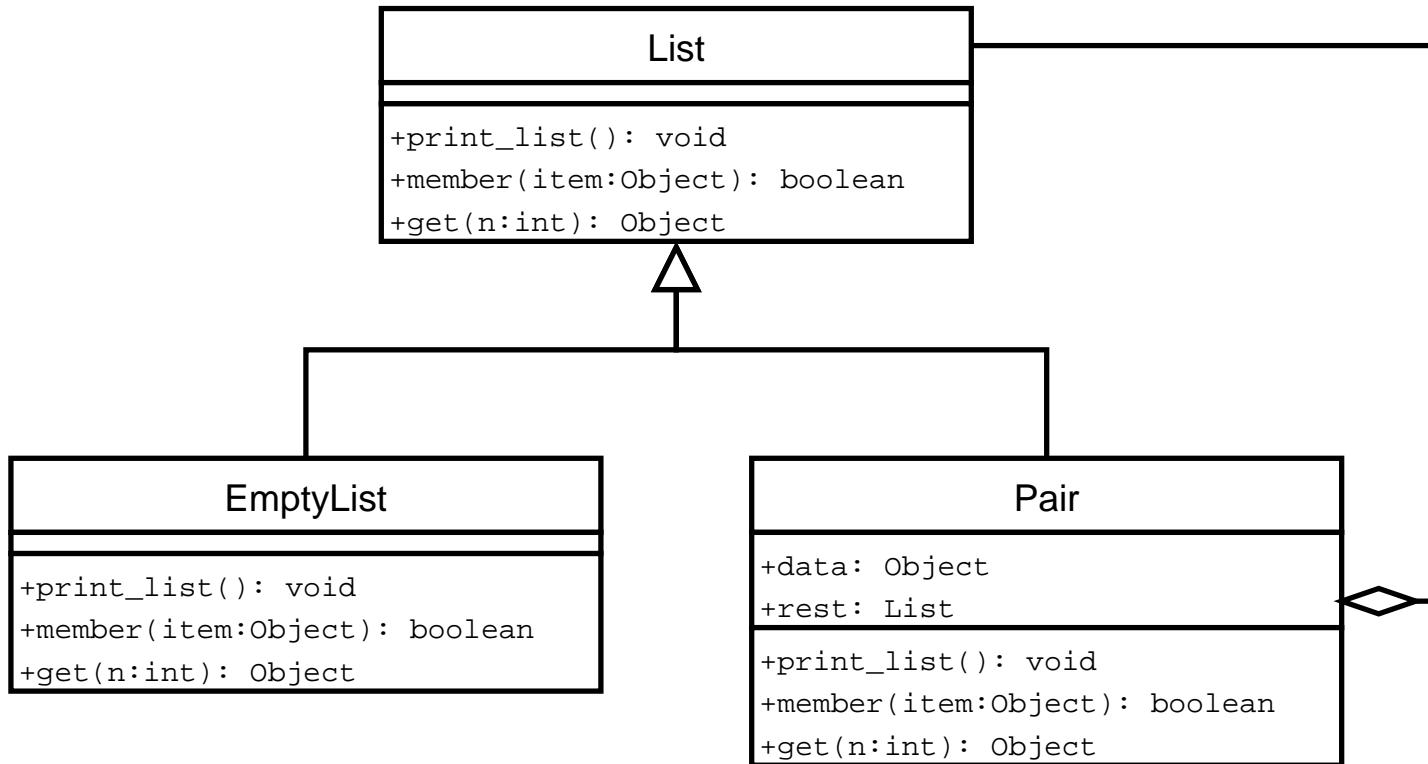
Design principle:

Sometimes a set of classes can be simplified by pushing responsibilities from a class to its subclasses

Recursive data-structures



Recursive data-structures



Recursive data-structures

```
class List
{
    void print_list()
    { ... }
    boolean member(Object item)
    { ... }
    Object get(int n)
    { ... }
}
```

Recursive data-structures

```
abstract class List
{
    abstract void print_list();

    abstract boolean member(Object item);

    abstract Object get(int n);
}
```

Recursive data-structures

```
class EmptyList extends List
{
}
```

Recursive data-structures

```
class EmptyList extends List
{
    void print_list()
    { ... }
    boolean member(Object item)
    { ... }
    Object get(int n)
    { ... }
}
```

Recursive data-structures

```
class EmptyList extends List
{
    void print_list()
    {
        System.out.print("");
    }
    boolean member(Object item)
    {
        return false;
    }
    Object get(int n)
    throws NoSuchElementException
    {
        throw new NoSuchElementException();
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    Pair(Object d, List l) { ... }

    Object getData() { return data; }
    List  getRest() { return rest; }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    Pair(Object d, List l) { ... }

    Object getData() { return data; }
    List  getRest() { return rest; }

    void print_list()
    { ... }
    boolean member(Object item)
    { ... }
    Object get(int n)
    { ... }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...

    void print_list()
    {
        Pair p = (Pair)this;
        Object data = p.getData();
        List rest = p.getRest();
        System.out.print(data + ", ");
        rest.print_list();
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...
    void print_list()
    {
        System.out.print(data + ", ");
        rest.print_list();
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...
    void print_list()
    {
        System.out.print(data + ", ");
        rest.print_list(); // Dynamic-dispatch
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...

    boolean member(Object item)
    {
        Pair p = (Pair) this;
        Object data = p.getData();
        if (data.equals(item))
        {
            return true;
        }
        else
        {
            List rest = p.getRest();
            return rest.member(item);
        }
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...

    boolean member(Object item)
    {
        if (data.equals(item))
        {
            return true;
        }
        else
        {
            return rest.member(item);
        }
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...
    ...

    boolean member(Object item)
    {
        if (data.equals(item))
        {
            return true;
        }
        else
        {
            return rest.member(item); // Dynamic-dispatch
        }
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...
    ...

    Object get(int n) throws NoSuchElementException
    {
        Pair p = (Pair)this;
        if (n == 0)
        {
            Object data = p.getData();
            return data;
        }
        else
        {
            List rest = p.getRest();
            return rest.get(n - 1);
        }
    }
}
```

Recursive data-structures

```
class Pair extends List
{
    Object data;
    List   rest;

    ...

    Object get(int n) throws NoSuchElementException
    {
        if (n == 0)
        {
            return data;
        }
        else
        {
            return rest.get(n - 1);
        }
    }
}
```

Recursive data-structures

- Terminology:
 - Linked-list: List
 - Node: Pair
 - Next: Rest

Recursive data-structures

- Alternative implementations of linked-lists:
 - Use constant `null` instead of class `EmptyList`
 - Class list has attributes:
 - * First node (front)
 - * Last node (tail)
 - Doubly-linked-lists:
 - * Each node has a reference to the *next* and the *previous* node
 - Circular linked-lists:
 - * The next of the last node is the first node

Recursive data-structures

- Useful applications of linked-lists:
 - Queues
 - Stacks

Recursive data-structures

- Useful applications of linked-lists:
 - Queues: (FIFO or LILO)
 - * Elements can be removed only from the front
 - * Elements can be added only at the end
 - Stacks: (LIFO or FILO)
 - * Elements can be removed only from the front (top)
 - * Elements can be added only at the front (top)

Recursive data-structures

Other recursive data-structures:

- Trees:
 - A tree is either:
 - * A node with two subtrees, or
 - * A leaf (a node with no subtrees)

What this course is about

- This course is an introduction to *computer programming*
- Computer programming: solving problems involving information by means of a computer

What this course is *not* about

- This course is *not* about...
 - ...how to use a computer
 - ...how to use software applications
 - ...how to use the Operating System
 - ...how to send e-mail
 - ...how to surf the Web
 - ...how to create Web pages
 - ...how to fix your printer
 - ...how to become a hacker
 - ...how to manage a computer system (installing software, fixing problems, etc.)
- There is no course in Computer Science about how to use computers, in the same way that there is no course in Mechanical Engineering that teaches how to drive a car or operate some machinery.

Objectives

- To learn:
 - ...a methodology to understand and solve problems involving information
 - ...how to think computationally
 - ...how to create simple algorithms
 - ...how to design and implement computer programs using the Java programming language
 - ...how to solve problems in an Object-Oriented manner
- This is neither a “computers course” nor a “Java course.”

Why is computer programming useful

- General benefits
 - Introduces a structured way of thinking, analysing and solving problems
- Applications
 - Engineering and Physical sciences: modelling and simulation
 - Biological sciences: Bioinformatics, Eco-system modelling
 - Geography, Environmental Studies and Urbanism: Geographic Information Systems
 - Economics: Economic forecasting and analysis, Economic modelling
 - Management: Databases, Information Systems, Process optimization
 - Software development

Computer Science

“Computer Science is no more about computers than Astronomy is about telescopes” - Edsger Dijkstra

- Computer Science:
 - Theory
 - Building systems

Computer Science

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- Computer Science:
 - Theory
 - Algorithms
 - Building systems

Computer Science

“Computer Science is no more about computers than Astronomy is about telescopes” - Edsger Dijkstra

- Computer Science:
 - Theory
 - Algorithms: studies algorithms in the abstract
 - * Algorithm design techniques
 - * Complexity analysis
 - * Data-structures
 - * etc.
 - Building systems

Computer Science

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- Computer Science:
 - Theory: studies the mathematical foundations of computation
 - Algorithms
 - Building systems

Computer Science

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- Computer Science:
 - Theory: studies the mathematical foundations of computation
 - * Theory of Computation: Automata and Formal languages
 - * Complexity Theory
 - * Semantics
 - Algorithms
 - Building systems

Computer Science

“Computer Science is no more about computers than Astronomy is about telescopes” - Edsger Dijkstra

- Computer Science:
 - Theory: studies the mathematical foundations of computation
 - Algorithms
 - Building systems
 - * Computer Graphics
 - * Robotics
 - * Artificial Intelligence
 - * Distributed Processing
 - * Operating Systems
 - * Modelling and Simulation
 - * Numerical computation
 - * Computer games
 - * Databases
 - * ...etc.

The end