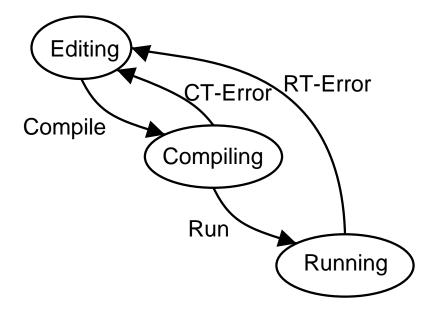
Errors

- Errors:
 - Compile-time errors
 - Run-time errors
 - * Exceptions
 - * Logical



Errors



- A Java program is made up of one or more class definitions
- A class definition is made up of zero or more method definitions
- A method definition is made up of zero or more statements and variable declarations
- Roles:
 - Classes: Modules and Types of objects
 - Methods: procedures, functions, algorithms
 - Statements: instructions



```
public class ClassName
{
    // Body of ClassName
    // ...
    // List of method definitions
}
```

```
public class HelloWorld
{
    // Body of ClassName
    // ...
    // List of method definitions
}
```

```
public class Classname
{
    // method header
    {
        // method body: list of statements
    }
}
```

```
public class HelloWorld
{
    public static void main(String[] args)
    {
        System.out.println("Hello");
        System.out.println("Good bye");
    }
}
```

Bad Java Syntax

```
public class HelloWorld
{
        System.out.println("Hello");
        System.out.println("Good bye");
}
```



Bad Java Syntax

```
public static void main(String[] args)
{
        System.out.println("Hello");
        System.out.println("Good bye");
}
```

Bad Java Syntax

```
public static void main(String[] args)
{
    public class HelloWorld
    {
        System.out.println("Hello");
        System.out.println("Good bye");
    }
}
```

```
public class HelloWorld
{
    public static void main(String[] args)
    {
        System.out.println("Hello");
        System.out.println("Good bye");
    }
}
```

```
public class HelloWorld
{
public static void main(String[] args)
{
System.out.println("Hello");
System.out.println("Good bye");
}
}
```

public class HelloWorld{public static void main(St ring[] args){System.out.println("Hello");System.ou t.println("Good bye");}}



User Interface

- The user interface of a program is the way it interacts with the user: keyboard/mouse/windows/text
- Graphical User Interface:
 - Windows: buttons, text boxes, slidebars, graphics, etc.
 - Input with mouse and keyboard.
- Textual User Interface:
 - Console window: plain text
 - Input: keyboard only
 - Output:

```
System.out.println("text");
```



Introduction to statements

• The print statement

```
System.out.println(string_literal);
System.out.print(string_literal);
```

• String literals:

```
"(almost) any characters"

"This is a string literal"

"String literals can contain almost any character,

"a"

"24"
```

Introduction to statements

• String concatenation:

```
string_literal + string_literal
string_literal + number_literal

'This is a "+"message"

'This is a message"

'There are "+70+" students in this class"
```

• String literals with numbers are not numbers: '17' is not the same as 17

is

while

$$17 + 29$$

is

46

Simple programs

```
// File: PrintingStuff.java
public class PrintingStuff
{
    public static void main(String[] args)
    {
        System.out.println("This trivial program journeys system.out.println("prints this text to a system.out.println("Window.");
    }
}
```

- A variable is a memory location
- A variable can contain information
- A variable has a symbolic name

age	
-----	--



age 20



\	,	•		
\/	ar	1	\mathbf{h}	lΩC
V	aı	ıa	U	にころ

last_name	
-----------	--

age

GPA

last_name "Smith"

age 20

GPA 3.5

last_name "Smith"

age 21

GPA 3.7



Variable declaration

- A variable declaration is a statement that declares that a variable is going to be used.
- A variable declaration goes inside some method
- A variable declaration has the form:

```
type identifier;
```

• Examples:

```
String last_name;
int age;
float GPA;
```



Assignment

- An assignment is a statement that gives a value to a variable
- An assignment goes inside some method
- An assignment has the form:

```
variable = value;
```

- Its meaning it to put the value into the memory location of the variable
- Examples:

```
last_name = "Smith";
age = 20;
```

• Note that the following are *incorrect*:

```
20 = age;
"Smith" = last_name;
```



Assignment

• The variable must be declared before being assigned a value

```
String last_name;
last_name = "Smith";
```

• But the following is wrong:

```
age = 20;
int age;
```

• The type of the value must be the same as the type of the variable

```
last_name = 20; // Incorrect
age = "Smith"; // Incorrect
```



Variables and String expressions

Variables can be used with concatenation in String expressions

• is equivalent to

```
"your age is 19"
```

• if the variable age contains the value 19

A simple program

```
public class PrintData
{
   public static void main(String[] args)
   {
      String last_name;
      int age;
      last_name = "Smith";
      age = 20;
      System.out.println("Your last name is " + last_name);
      System.out.println("You are " + age + " years old");
   }
}
```

Basic java programs

```
public class ClassName
{
    public static void main(String[] args)
    {
        // Statements
    }
}
```

Statements

• Print statement

```
System.out.println(string_expression);
```

Variable declaration

```
type identifier;
```

Assignment

```
variable = value;
```

• Statements in a method are executed in *sequential order* from top to bottom

Assignment

• In an assignment

```
variable = value;
```

• the variable must have been declared before,

```
x = 7; // incorrect
int x;
```

• the type of the variable must match the type of the value

```
int x;
x = "7"; // incorrect
```

Sequential execution

```
public class OrderTest
{
    public static void main(String[] args)
    {
        int a;
        int b;
        a = 2;
        b = 3;
        b = 5;
        a = 8;
        System.out.println(a);
        System.out.println(b);
    }
}
```

Sequential execution

```
public class OrderTest
{
    public static void main(String[] args)
    {
        int a;
        int b;
        b = 5;
        a = 8;
        a = 2;
        b = 3;
        System.out.println(a);
        System.out.println(b);
    }
}
```

Some syntactic shortcuts

• Several variables of the same type can be declared in the same variable declaration:

```
type var1, var2, ..., varn;
```

• Examples:

```
int a;
int b;
```

is equivalent to

```
int a, b;
```

Some syntactic shortcuts

A variable can be initialized when declared

```
int a;
a = 2;
```

is equivalent to

int
$$a = 2$$
;

• But a variable cannot be redeclared, so

is incorrect, while the following is correct



User Interface

- Interaction between the user and some program
- Textual UI
 - Output:

```
System.out.println(string_expression);
```

– Input:

```
scanner.nextInt();
scanner.nextLine();
```

• Examples:

```
Scanner myScanner = new Scanner(System.in);
int n;
n = myScanner.nextInt();
```



User Interface

```
import java.util.Scanner;
public class UserInputTest {
    public static void main(String[] args)
        Scanner myScanner = new Scanner(System.in);
        String name;
        int age;
        System.out.print('Enter your name: ');
        name = myScanner.nextLine();
        System.out.print('Enter your age: ');
        age = myScanner.nextInt();
        System.out.println("Your name is " + name);
        System.out.println("You are " + age + " years old");
    }
```

}



Data types

• Each variable has a data type

```
String major;
int age;
```

- A data type is a set of possible values
 - int is the set of integers
 - String is the set of strings
 - float is the set of rational numbers written as a decimal expansion
 - double is the set of rational numbers as a decimal expansion, with double precision
 - char is the set of characters
 - boolean is the set {true, false}
 - byte is the set of bytes, writen in decimal



Data types

Data type	Possible values	Examples	
int	all integers between -2^{31} and $2^{31}-1$	0, 1, 2, -3, -1729	
String	all character strings enclosed in ""	''hello bye'', '''', ''a''	
float	rationals between $-3.4 imes 10^{38}$ and $3.4 imes 10^{38}$	0.0f, -2.3f, 111.001f	
double	rationals between $-1.7 imes 10^{308}$ and $1.7 imes 10^{308}$	0.0, -2.3, 111.001	
char	all individual characters enclosed in ''	'a', 'b','z','7','+','A'	
boolean	only true and false	true, false	
byte	all integers between -128 and 127	-128, 0, 8	
long	integers between -2^{63} and $2^{63}-1$	0l, 65536l, -3l	
short	integers between -2^{15} and $2^{15}-1$	-3, -2, 0, 1, 4	



Data types

Data type	Size	Range
boolean	8 bits (7 unused)	0 - 1
byte	8 bits	-2^7 to 2^7-1
char	8 bits (ASCII), 16 bits (Unicode)	0 to 2^8 (ASCII) 0 to 2^{16} (Unicode)
short	16 bits	-2^{15} to $2^{15}-1$
int	32 bits	-2^{31} to $2^{31}-1$
long	64 bits	-2^{63} to $2^{63}-1$



Real numbers

$$\sqrt{2}$$

$$\sqrt{2}$$
 $\sqrt{3}$

 π

$$e = 2.718...$$

$$\varphi = \frac{1 \pm \sqrt{5}}{2} = \begin{cases} 1.618... \\ 0.618... \end{cases}$$

Assignment and data types

Arithmetic expressions

• If a variable is of a numeric type (int, float, long, etc.) then an assignment can take the form

```
variable = arithmetic_expression;
```

- where arithmetic_expression is an expression involving:
 - numbers (of the appropriate type)
 - operators (+, -, *, /, %)
 - variables (of numeric type)
 - parenthesis
- Example:



Arithmetic expressions

Parenthesis are used to group operations:

```
double grade, assignments, midterm, final;
  double a1 = 20, a2 = 19, a3 = 9, a4 = 14, a5 = 18;
  assignments = a1 + a2 + a3 + a4 + a5;
  midterm = 75.5;
  final = 80.0;
  grade = assignments * 0.25
        + midterm * 0.20
        + final * 0.55;
is equivalent to
  double grade, midterm, final;
  double a1 = 20, a2 = 19, a3 = 9, a4 = 14, a5 = 18;
  midterm = 75.5;
  final = 80.0;
  grade = (a1 + a2 + a3 + a4 + a5) * 0.25
        + midterm * 0.20
        + final * 0.55;
```

Operator precedence

Operators are evaluated depending on their precedence:

result =
$$6 + 5 * 3$$
;

- If the operators did not have precedence, the expression would have as value 33
- But it's real meaning is:

result =
$$6 + (5 * 3)$$
;

- Which evaluates to 6 + 15 which is 21.
- Operators have "associativity":

result =
$$6 + 5 + 3 + 9$$
;

• is evaluated as

result =
$$((6 + 5) + 3) + 9$$
;

Operator precedence

Precedence level	Operator	Operation	Associativity	
1	+	unary plus	right to left	
1	-	unary minus	I fight to left	
	*	multiplication		
2	/	division	left to right	
	%	remainder (modulo)		
	+	addition		
3	-	substraction	left to right	
	+	string concatenation		



Operator precedence

$$r = 8 / 2 / 2;$$

is evaluated as $r = ((8 / 2) / 2);$

and

$$s = 12 * 2 - - 3;$$

is evaluated as

$$s = (12 * 2) - (-3);$$

and

$$t = -2 * 4 + - (a - 1);$$

is evaluated as

$$t = ((-2) * 4) + (- (a - 1));$$

Precedence

$$a + b + c + d + e$$
 $1 2 3 4$

is the same as (a+(b*c))-(d/e)

$$(a + b) * (c - d) / e$$

1 3 2 4

```
double a, b;
a = 2.0;
b = a;
a = 3.0;
System.out.println(a);
System.out.println(b);
```

```
double a, b;
a = 2.0;
b = a;
a = 3.0;
System.out.println(a);
System.out.println(b);
// Prints
// 3.0
// 2.0
```

a

b

a 2.0

b

a 2.0

b 2.0

a 3.0

b 2.0

```
double a, b;
a = 2.0;
a = 3.0;
b = a;
System.out.println(a);
System.out.println(b);
```



```
double a, b;
a = 2.0;
a = 3.0;
b = a;
System.out.println(a);
System.out.println(b);
// Prints
// 3.0
// 3.0
//
// a and b have the same contents (the same value)
// but they are different variables
```

a 2.0

b

a 3.0

b

a 3.0

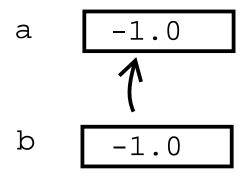
b 3.0

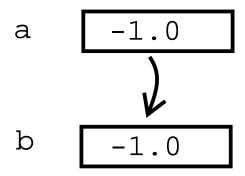
```
double a, b;
a = 2.0;
b = -1.0;
a = b;
b = a;
System.out.println(a);
System.out.println(b);
```

```
double a, b;
a = 2.0;
b = -1.0;
a = b;
b = a;
System.out.println(a);
System.out.println(b);
// Prints
// -1.0
// -1.0
```

a 2.0

b -1.0





```
double a, b, c;
a = 2.0;
b = -1.0;
c = a;
a = b;
b = c;
System.out.println(a);
System.out.println(b);
```

```
double a, b, c;
a = 2.0;
b = -1.0;
c = a;
a = b;
b = c;
System.out.println(a);
System.out.println(b);
// Prints
// -1.0
// 2.0
// This implements a 'swap' between variables
```

a

C

b

a 2.0

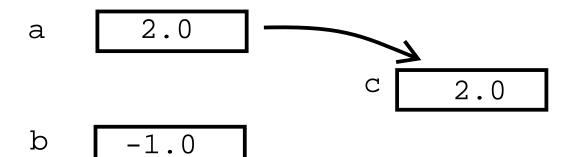
С

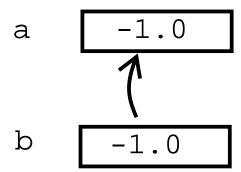
b

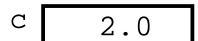
a 2.0

С

b -1.0







a -1.0

b 2.0

ORDER MATTERS



More on arithmetic operations

- The remainder operator % computes the remainder of an integer division (not percentages!)
 - 8 % 2 evaluates to 0
 - -7% 2 evaluates to 1
 - 8 % 3 evaluates to 2
 - -3% 5 evaluates to 5
- In general, for every integers a and b, $0 \leqslant a\%b < b$
- Division has a different meaning for integers and for floats and doubles
- The division between two integers is an integer
- The division between a float or double and an integer is a float or double
 - -8/3 evaluates to 2
 - 8.0 / 3 evaluates to 2.666666...



Statements

• Variable declaration

```
type identifier;
```

Assignment

```
variable = expression;
```

• User Interface: output

```
System.out.println(string_expression);
```



• User Interface: input

```
variable = scanner.nextType();
```



Primitive Data Types

General category	Туре	Description	Examples
Numeric	int	Integers	0,1,-3
	long	Long integers	65537l
	short	Short integers	2,-6
	byte	Bytes	255
	float	Rationals	1.33f
	double	Rationals	1.618
Text	char	Single characters	'x', '''
	String	Sequences of characters	''abc''
Logic	boolean	Truth values	true, false



Syntax of string expressions

- A *string expression e* is either
 - a string literal (characters enclosed in " ")
 - or a variable of String type
 - or an expression of the form

$$e_1 + e_2$$

where e_1 is a string expression, + is the concatenation operator, and e_2 is either a string expression or an arithmetic expression or a character expression

Examples:

```
"this is a long literal"
"my name is " + name
"the average is " + (a + b)/2
"I am taking " + n + " courses"
"My initials are " + 'E' + 'P'
```



```
public class SomeProgram {
   public static void main(String[] args)
   {
      // List of statements
   }
}
```



```
public class SomeProgram {
   public static void main(String[] args)
   {
     int n = 4;
     ''I am taking '' + n + '' courses''; // WRONG!
   }
}
```

```
public class SomeProgram {
   public static void main(String[] args)
   {
     int n = 4;
     System.out.println("I am taking" + n + " courses");
   }
}
```

```
public class SomeProgram {
   public static void main(String[] args)
   {
     int n = 4;
     String message = "I am taking" + n + " courses";
   }
}
```

```
public class SomeProgram {
   public static void main(String[] args)
   {
     float a = 4, b = 2, c;
     c = (a + b) / 2;
   }
}
```

Statements vs expressionfs

- A statement is not an expression
- An expression is not a statement
- An expression is a term (e.g. x*2, "a"+b, etc.) inside a statement, which has a value.
- A statement is an instruction to be executed (e.g. assignment, print, etc.) and it has no value.
- A method's body is a list of statements, not a list of expressions



Strings

```
String first_name, last_name, temp;
first_name = "Adam";
last_name = Smith;
System.out.println(first_name);
System.out.println(last_name);
```



Strings

```
String first_name, last_name, temp;
first_name = "Adam";
last_name = Smith;
System.out.println(first_name);
System.out.println(last_name);
```



Strings

```
String first_name, last_name, temp;
first_name = "Adam";
last_name = "Smith";
System.out.println(first_name);
System.out.println(last_name);
```



```
String first_name, last_name;
first_name = "Adam";
last_name = "Smith";
last_name = first_name;
first_name = last_name;
System.out.println(first_name);
System.out.println(last_name);
```



Adam Adam



```
String first_name, last_name, temp;
first_name = "Adam";
last_name = "Smith";
temp = last_name;
last_name = first_name;
first_name = temp;
System.out.println(first_name);
System.out.println(last_name);
```



Smith Adam



```
String first_name, last_name, temp;
first_name = "Adam";
last_name = "Smith";
temp = last_name;
last_name = first_name;
first_name = temp;
System.out.println(first_name);
System.out.println(last_name);
```

```
String first_name, last_name, temp;
first_name = "Adam";
last_name = "Smith";
temp = last_name;
first_name = temp;
last_name = first_name;
System.out.println(first_name);
System.out.println(last_name);
```

Smith Smith



Assignment

- Assignment is not equality
- The right-hand side of an assignment can contain the same variable as the left hand-side:

```
int count = 0;
// Here the value of count is 0
count = count + 1;
// Here the value of count is 1
String name;
name = "Bond";
name = "James" + name;
```

String concatenation is not commutative (a+b is not b+a)

Operators and types

 The meaning of an operator depends on its context, and in particular on the types of its arguments

```
int a = 8, b = 3, c;
c = a / b; // Integer division

double d = 8.0, e = 3.0, f;
f = d / e; // Floating point division

int g;
g = a + b; // Addition

String h = "one", i = "two", j;
j = h + i; // String concatenation
```

Assignment

• If variable is of numeric type (int, float, etc.)

```
variable = arithmetic_expression;
```

• If variable is of String type

```
variable = string_expression;
```

• If variable is of boolean type

```
variable = boolean_expression;
```

Boolean expressions

```
true
false
true && true
false || true
!false
!true && false || !false
!(sunny && false)
5 < 7
6 >= 8
2 + x == 9 \&\& b < 8 \mid \mid c == true
```

Syntax of boolean expressions

- ullet A boolean expression e is either
 - the constant true
 - or the constant false
 - or a boolean variable
 - or an expression of the form

$$e_1$$
 boolop e_2

where e_1 and e_2 are boolean expressions and boolop is one of the binary boolean operators: && (and) or || (or)

or an expression of the form

!e'

where e' is an boolean expression and ! is the unary boolean operator for negation.

or an expression of the form

(e')

where e^{\prime} is an boolean expression

or an expression of the form

$$e_1$$
 relop e_2

where e_1 and e_2 are arithmetic expressions and relop is one of the binary relational operators: <, <=, ==, >



Boolean expressions

```
boolean a, b;
a = true;
b = !a;

int x, y, d;
boolean c;
c = x - y >= 0 && x - y < d;
c = ((x - y) >= 0) && ((x - y) < d);

float temp = -25.2f, windchill = -35.2f;
boolean sunny = true, rain, windy, cold, ski;
rain = !sunny;
windy = windchill - temp > -10.0f;
cold = temp < -20.0f;
ski = sunny && !windy || !cold;</pre>
```

Boolean expressions

```
float temp = -25.2f, windchill = -35.2f;
boolean sunny = true, rain, windy, cold, ski;
rain = !sunny;
windy = windchill - temp > -10.0f;
cold = temp < -20.0f;
ski = sunny && !windy || !cold;
rain = true;

boolean b = true;
b = false;
b = !b;
b = true && false;</pre>
```

Precedence

Precedence	Operator	Operation	Associativity	
1	+	Unary plus		
	-	Unary minus	right to left	
	ļ.	Logical negation (NOT)		
2	*	Multiplication	left to right	
	/	Division		
	%	Remainder (modulo)		
3	+	Addition	left to right	
	-	Substraction		
	+	String concatenation		
4	<	Less than	Left to right	
	<=	Less than or equal to		
	>	Greater than		
	>=	Greater than or equal to		
5	==	Equals to	Left to right	
	!=	Different to		
6	&&	Logical conjunction (AND)	Left to right	
7		Logical disjunction (OR)	Left to right	



Precedence

is the same as (((4+x)==9)&&(b<8))||(!c)

is the same as (((4+x)==9)&&(b<8))||(!(1<x))

Semantics of expressions

- The meaning of an expression is the value of the expression
 - An arithmetic expression is evaluated to a number
 - A string expression is evaluated to a string
 - A boolean expression is evaluated to a truth-value (true or false)



Semantics of boolean expressions

- The value of true is true
- The value of false is false
- The value of a boolean variable is whatever value it contains
- The value of $e_1\&\&e_2$ is true if the values of e_1 and e_2 are both true, and false otherwise
- The value of $e_1||e_2|$ is true if the value of e_1 or the value of e_2 true, and false if the values of both are false
- ullet The value of !e is true if the value of e is false, and false if the value of e is true



- ullet The value of $a_1 < a_2$ is true if the value of a_1 is strictly less than the value of a_2
- The value of $a_1 <= a_2$ is true if the value of a_1 is less or equal to the value of a_2
- The value of $a_1 > a_2$ is true if the value of a_1 is strictly greater than the value of a_2
- ullet The value of $a_1>=a_2$ is true if the value of a_1 is greater or equal to the value of a_2
- ullet The value of $a_1 == a_2$ is true if the value of a_1 is equal to the value of a_2
- ullet The value of $a_1!=a_2$ is true if the value of a_1 is different to the value of a_2



- Truth tables
- Assume that a and b are boolean expressions

а	!a
true	false
false	true

а	b	a && b
true	true	true
true	false	false
false	true	false
false	false	false

а	b	a b	
true	true	true	
true	false true		
false	true	true	
false	false	false	



true && false || !false

is the same as the value of

 (true && false) || (!false)

which is

 (true && false) || true

which is

• The value of

false || true

which is

true



• Exclusive or: either a is true or b is true but not both

a && !b || !a && b

а	b	!b	a&&!b	!a	!a&&b	a&&!b !a&&b
true	true	false	false	false	false	false
true	false	true	true	false	false	true
false	true	false	false	true	true	true
false	false	true	false	true	false	false

• What is the value of 4+x==9 && b < 8 || !c ?

- What is the value of 4+x==9 && b < 8 || !c ?
- It depends on the values of the relational expressions (4+x==9), (b<8) and the boolean expression c.
- These expressions depend on the values of x, b and c, which we do not know
- but we can consider the all the possible truth values for each subexpression:

4+x==9	b<8	С	!c	4+x==9 && b<8	(((4+x)==9)&&(b<8)) (!c)
true	true	true	false	true	true
true	true	false	true	true	true
true	false	true	false	false	false
true	false	false	true	false	true
false	true	true	false	false	false
false	true	false	true	false	true
false	false	true	false	false	false
false	false	false	true	false	true



temp $> -20.0 \parallel$!windy && sunny

temp>-20.0	windy	sunny	!windy	!windy && sunny	(temp > -20.0) (!windy && sunny)
true	true	true	false	false	true
true	true	false	false	false	true
true	false	true	true	true	true
true	false	false	true	false	true
false	true	true	false	false	false
false	true	false	false	false	false
false	false	true	true	true	true
false	false	false	true	false	false



Note about variables

- The name of a variable is just a symbolic name to make the program more readable
- The name of the variable does not give the variable any special meaning

```
double temp = -27.0;
boolean cold;
cold = temp <= -20.0;</pre>
```

- Does not mean that it is actually cold!
- It only means

```
double x = -27.0;
boolean y;
y = x <= -20.0;</pre>
```

 But it is useful for the programmer to give variables meaningful names

- Sometimes it is useful to look at data as if they were from a different type
- For example:
 - Adding an integer and a double
 - Obtaining the ASCII code of a character
- Forms of data conversion:
 - Implicit:
 - * Assignment conversion
 - * Promotion
 - Explicit: Casting



 Assignment conversion: A value of one type is assigned to a variable of a different type, as long as the types are compatible

```
int n = 7;
double d = n;
long k = n;
int m = d; // Wrong: compile-time error
```

 Promotion: an expression "promotes" the types of its operands to its "largest" type

```
int m = 8;
float x = 3.0f, y;
y = x + m;
```

Casting expressions (not a statement)

```
(type)expression
```

• Examples:

```
int n = 3;
double p;
p = (double)n + 4.0;

int a = 3, b = 8;
float c, d;
c = b/a;
d = (float)b/a;
System.out.println(c); // 2.0
System.out.println(d); // 2.666666...
```

```
double r = 2.41;
int a;
a = r; // Error
```

```
double r = 2.41;
int a;
a = (int)r; //OK: Narrowing casting
```

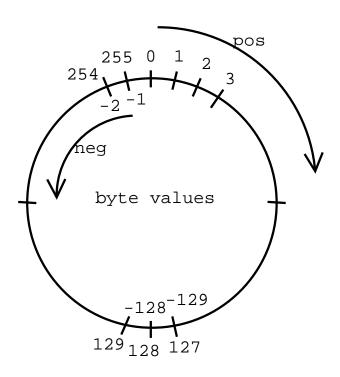
- There are two types of casting:
 - Narrowing conversions: from a type which requires more memory to a type that requires less
 - Widening conversions: from a type which requires less memory to a type which requires more
- If expression has type t, and t requires more memory than type s, then (s) expression is a narrowing conversion (e.g. int to byte, double to float, float to int, ...)
- If expression has type t, and t requires less memory than type s, then (s) expression is a widening conversion (e.g. byte to double, long to int, ...)



- Widening conversions are safe: no loss of information
- Narrowing conversions are not safe: possible loss of information

```
float x = 2.71f;
int i = (int)x;
// i == 2

int k = 130;
byte b = (byte)k;
// b = -126
```



$$128 = -128$$

 $129 = -127$
 $256 = 0$
 $257 = 1$

byte b
$$\begin{array}{ccc} & \mathrm{b} + k2^8 = \mathrm{b} \\ & \mathrm{int} \ \mathrm{i} \\ & \mathrm{k} \ \mathrm{is} \ \mathrm{any} \ \mathrm{integer} \end{array}$$
 k is any integer

The end

