Computer Architecture: Adders

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Addition

$$
\begin{array}{r}
00101110 \\
+\ 00100111 \\
\hline
\end{array}
$$

Addition

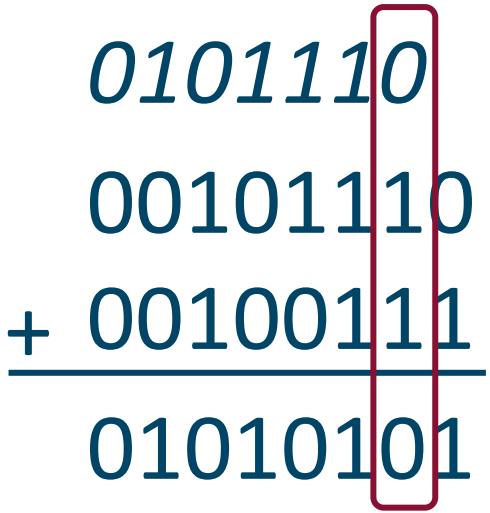
0101110

00101110

\+ 00100111

01010101

Addition



Addition

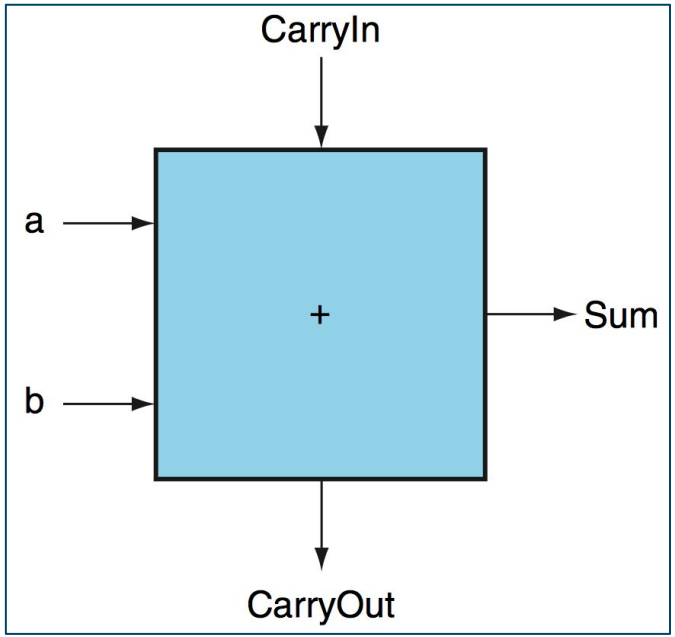
Inputs			Outputs		Comments
a	b	CarryIn	CarryOut	Sum	
0	0	0	0	0	$0 + 0 + 0 = 00_{two}$
0	0	1	0	1	$0 + 0 + 1 = 01_{two}$
0	1	0	0	1	$0 + 1 + 0 = 01_{two}$
0	1	1	1	0	$0 + 1 + 1 = 10_{two}$
1	0	0	0	1	$1 + 0 + 0 = 01_{two}$
1	0	1	1	0	$1 + 0 + 1 = 10_{two}$
1	1	0	1	0	$1 + 1 + 0 = 10_{two}$
1	1	1	1	1	$1 + 1 + 1 = 11_{two}$

1-Bit Full Adder

1-bit addition:

- CarryOut output is 1 when at least two inputs are 1

- Sum output is 1 when exactly one input is 1 or all three inputs are 1



1-Bit Full Adder

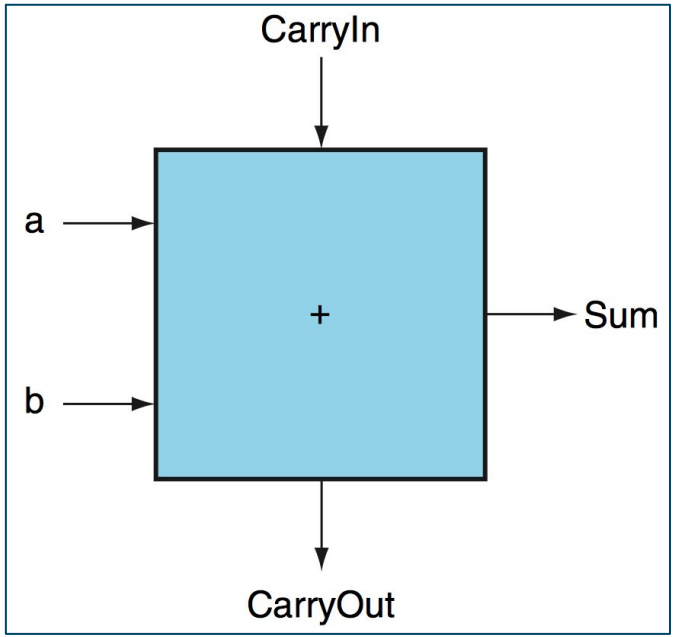
1-bit addition:

- CarryOut output is 1 when at least two inputs are 1

$$\text{CarryOut} = (b \cdot \text{CarryIn}) + (a \cdot \text{CarryIn}) + (a \cdot b)$$

- Sum output is 1 when exactly one input is 1 or all three inputs are 1

$$\text{Sum} = (a \cdot \overline{b} \cdot \overline{\text{CarryIn}}) + (\overline{a} \cdot b \cdot \overline{\text{CarryIn}}) + (\overline{a} \cdot \overline{b} \cdot \text{CarryIn}) + (a \cdot b \cdot \text{CarryIn})$$

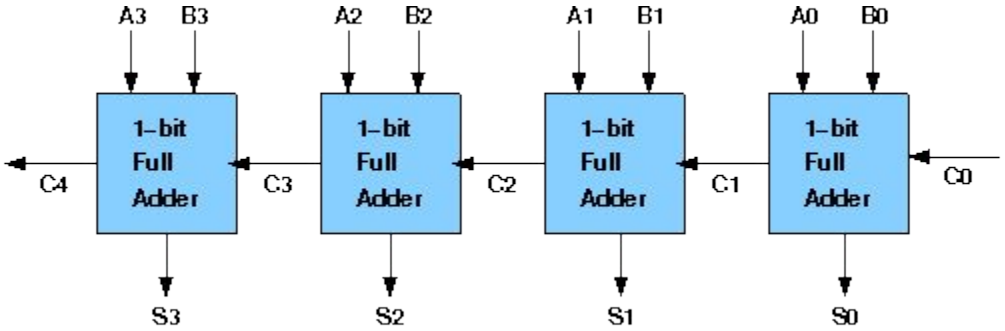


Ripple Carry Adder

n-bit addition:

- Series of 1-bit full adders

- Carry ripples through addition = Slow!



Calculate Carry

```
  ????  ????
  0010 1110
+ 0010 0111
-----------
```

Propagate and Generate

Generate: “When does a_i and b_i generate a carry?”

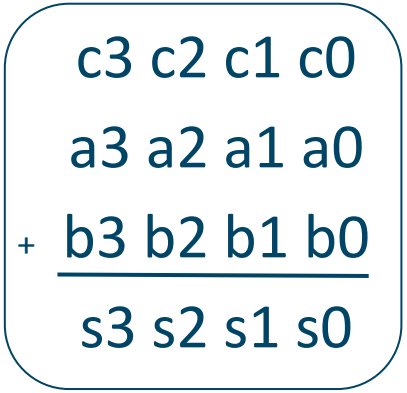
$g_i = a_i \cdot b_i$

Propagate: “When does a_i and b_i propagate a carry?”

$p_i = a_i + b_i$

CarryIn:

$c_{i+1} = g_i + p_i \cdot c_i$



Carry Lookahead Adder

$$c1 = g0 + (p0 \cdot c0)$$

$$c2 = g1 + (p1 \cdot g0) + (p1 \cdot p0 \cdot c0)$$

$$c3 = g2 + (p2 \cdot g1) + (p2 \cdot p1 \cdot g0) + (p2 \cdot p1 \cdot p0 \cdot c0)$$

$$c4 = g3 + (p3 \cdot g2) + (p3 \cdot p2 \cdot g1) + (p3 \cdot p2 \cdot p1 \cdot g0) + (p3 \cdot p2 \cdot p1 \cdot p0 \cdot c0)$$

$$\begin{array}{rr} & c3\ c2\ c1\ c0 \\ & a3\ a2\ a1\ a0 \\ + & b3\ b2\ b1\ b0 \\ \hline & s3\ s2\ s1\ s0 \end{array}$$

Carry Lookahead Adder

????	????
0010	1110
+0010	+ 0111

Super Propagates and -Generates

Calculate CarryIn of each 4-bit carry-lookahead adder.

Superpropagate Pi and supergenerate Gi :

$$P0 = p3 \cdot p2 \cdot p1 \cdot p0$$

$$G0 = g3 + (p3 \cdot g2) + (p3 \cdot p2 \cdot g1) + (p3 \cdot p2 \cdot p1 \cdot g0)$$

Calculate Ci :

$$C1 = G0 + (P0 \cdot c0)$$

$$C2 = G1 + (P1 \cdot G0) + (P1 \cdot P0 \cdot c0)$$

$$C3 = \ldots$$

8-bit Carry Lookahead Adder