Computer Systems and Architecture
Data Representation

Stephen Pauwels

Academic Year 2021-2022

Universiteit Antwerpen

Base 10 to Base 2

- Convert $(23.375)_{10}$ to base 2.

Base 10 to Base 2

- Convert $(23.375)_{10}$ to base 2.
  - Step 1. Remainder method: convert $(23)_{10}$ to base 2

	Remainder	
23 / 2 = 11	1	LSB
11 / 2 = 5	1	
5 / 2 = 2	1	
2 / 2 = 1	0	
1 / 2 = 0	1	MSB

- $(23)_{10} = (10111)_2$

Base 10 to Base 2

- Convert $(23.375)_{10}$ to base 2.
  - Step 1. Remainder method: convert $(23)_{10 \text{ to}}$ base 2
  - Step 2. Multiplication method: convert $(.375)_{10}$ to base 10

.375 x 2 = **0** .75

.75 x 2 = **1** .5

.5 x 2 = **1** .0

- $(.375)_{10} = (.011)_2$

Base 10 to Base 2

- Convert $(23.375)_{10}$ to base 2.
  - Step 1. Remainder method: convert $(23)_{10\ to}$ base 2
  - Step 2. Multiplication method: convert $(.375)_{10}$ to base 10
  - Step 3. Total: $(23.375)_{10}$
    - $(10111.011)_2$

Base 2 to Base 10

- Number in base 2:
  - $b_n b_{n-1} \ldots b_1 b_0 \, . \, b_{-1} b_{-2} \ldots b_{-m}$
- Value in base 10
  - $\sum_{i=-m}^{n-1} b_i * 2^i$
- Weighted Position Code
- Polynomial method

Base 2 to Base 10

- Convert $(1010.01)_2$ to base 10
- $(1010.01)_2$

$= (1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 0 * 2^0 + 0 * 2^{-1} + 1 * 2^{-2})_{10}$

$= (8 + 0 + 2 + 0 + 0 + .25)_{10}$

$= (10.25)_{10}$

Converting to other bases

- Conversion between base *k* and base 10
  - Remainder method
    - Divide by *k*
  - Multiplication method
    - Multiply by *k*
  - Polynomial method
    - $\sum_{i=-m}^{n-1} b_i * k^i$

Negative Numbers

- Signed Magnitude
  - First bit as sign
  - $(+12)_{10} = (00001100)_2$

    $(-12)_{10} = (10001100)_2$

Negative Numbers

- Signed Magnitude
- One's Complement
  - Convert all zero's in 1's and 1's in zero's
  - $(+12)_{10} = (00001100)_2$

    $(-12)_{10} = (11110011)_2$

Negative Numbers

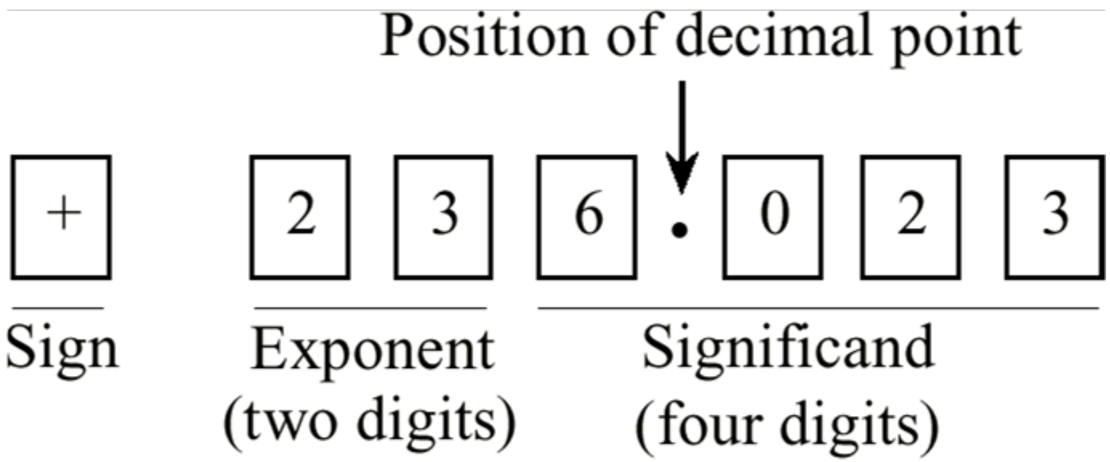
- Signed Magnitude
- One’s Complement
- Two’s Complement
  - Take the one’s complement, and add 1
  - $(+12)_{10} = (00001100)_2$
  - $(-12)_{10} = (11110100)_2$

Negative Numbers

- Signed Magnitude
- One's Complement
- Two's Complement
- Excess
  - Add bias to every number and convert it as a positive number
  - $(+12)_{10} = (100001100)_2$ $(12 + \mathit{128} = 140)$
    $(-12)_{10} = (01110100)_2$ $(-12 + \mathit{128} = 116)$

Floating Point

- Wide range of numbers in a limited number of bits
- Example: +6.023 * 10^{23}



Normalization

- Different representations for the same number
  - $254 * 10^0$

    $25.4 * 10^1$

    $0.0254 * 10^4$

    $254000 * 10^{-3}$
- Normalization: the dot is placed right of the leftmost digit
  - $2.54 * 10^2$
- Hidden bit: in base 2 the leftmost bit can be discarded
  - $1.1010 \rightarrow 1010$

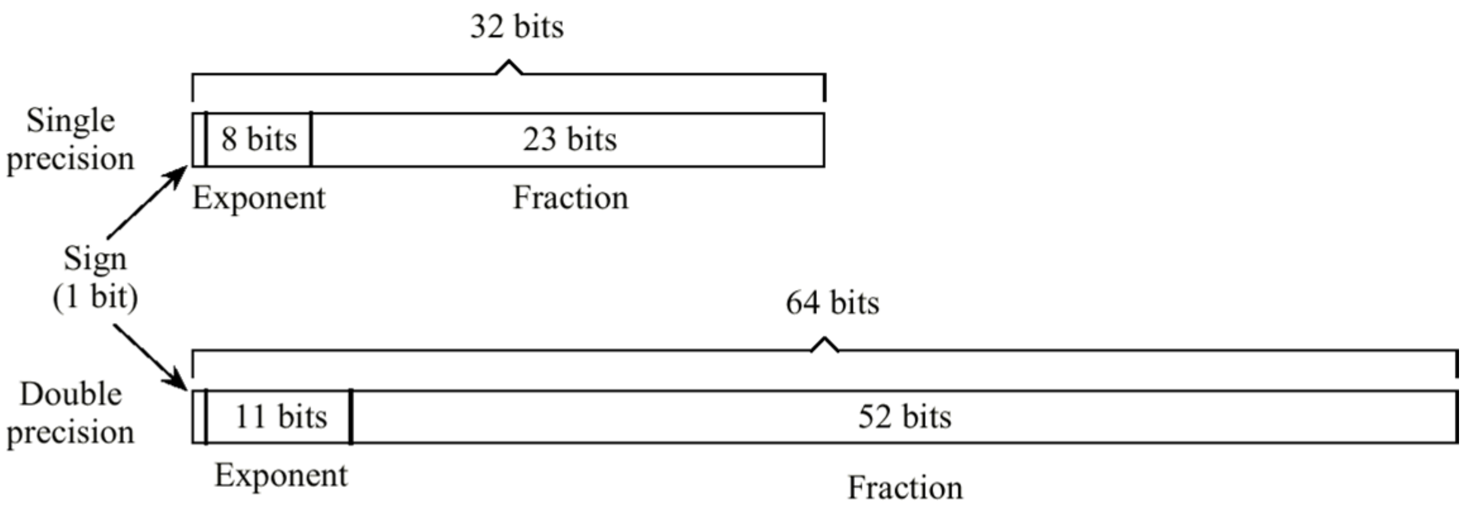
Conversion

- Convert $(9.375 * 10^{-2})_{10}$ to base 2 (scientific notation)

1. Fixed point: $(.09375)_{10}$
2. Multiplication method: $(.0011)_2$
3. Normalize: $.0011 = 0.0011 * 2^0 = 1.1 * 2^{-3}$

IEEE-754

- Sign bit
- Exponent: Excess 127 – 1023
- Number representation with hidden bit



IEEE-754

- Represent $(-12.625)^{10}$ in IEEE-754 Single Precision format

1. Convert to base 2
   - $(-12.625)_{10} = (-1100.101)_2$
2. Normalize
   - $(-1100.101)_2 = (-1.100101)_2 * 2^3$
3. Calculate exponent in excess 127
   - $3 + 127 = 130 \rightarrow (10000010)_2$

1 1000 0010 100 1010 0000 0000 0000 0000

IEEE-754 – Special Cases

- +0
  - 0 0000 0000 000 0000 0000 0000 0000 0000
- +∞
  - 0 1111 1111 000 0000 0000 0000 0000 0000
- + NaN
  - 0 1111 1111 001 0010 0000 0100 1000 0000
- 2^{-128} (Denormalized): $(1 * 2^{-2}) * 2^{-126}$
  - 0 0000 0000 010 0000 0000 0000 0000 0000

Exercises

- Blackboard
- Course webpage
  - http://msdl.uantwerpen.be/people/hv/teaching/ComputerSystemsArchitecture/#CS4