Statecharts aka Harel Charts

Visual Modelling

1. Higraph formalism

2. Statechart formalism (combines Higraphs and State Automata)

Diverse applications.
In particular: concurrent systems behaviour
Higraphs: Visualising Information

- complex
- non-quantitative, structural
- topological, not geometrical
- Euler
  - Venn diagrams (Jordan curve: inside/outside): enclosure, intersection
  - graphs (nodes, edges: binary relation); hypergraphs
Venn diagrams, Euler circles

- **topological** notions (syntax):
  - enclosure, exclusion, intersection

- Used to represent (denote) **mathematical** set operations:
  - union, difference, intersection
Hypergraphs

- **topological** notion (syntax): connectedness
- Used to represent (denote) relations between sets.
- Hyperedges: non longer binary relation ($\subseteq X \times X$):
  $\subseteq 2^X$ (undirected), $\subseteq 2^X \times 2^X$ (directed).
**Higraphs**: combining graphs and Venn diagrams

- sets + cartesian product
- hypergraphs
Blobs: set *inclusion*, *not membership*
Unique Blobs (atomic sets, no intersection)

- atomic blobs are identifiable sets
- other blobs are union of enclosed sets (e.g., $K = L \cup M \cup N \cup O \cup P$)
- empty space meaningless, identify intersection (e.g., $N = K \cap W$)
Unordered Cartesian Product: Orthogonal Components

\[ K = G \times H = H \times G = (L \cup M) \times (N \cup O \cup P) \]
Meaningless syntactic constructs

A

B

C

D
Induced Acyclic Graph (blob/orth comp alternation)
Adding (hyper) edges

- hyperedges
- attach to contour of any blob
- inter-level possible (e.g., denote global variables binding)
Clique Example
Clique: fully connected semantics
Entity Relationship Diagram (is-a)
Higraph version of E-R diagram
Extending the E-R diagram

- employees
  - men
  - women
  - secretaries
  - others
  - pilots

- works for
- paid on
- can fly
- married

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Modelling and Simulation: Higraphs/Statecharts
Formally (syntax)

A higraph $H$ is a quadruple

$$H = (B, E, \sigma, \pi)$$

$B$: finite set of all unique blobs

$E$: set of hyperedges

$$\subseteq X \times X, \quad \subseteq 2^X, \quad \subseteq 2^X \times 2^X$$

The subblob (direct descendants) function $\sigma$

$$\sigma : B \rightarrow 2^B$$

$$\sigma^0(x) = \{x\}, \quad \sigma^{i+1} = \bigcup_{y \in \sigma(i)} \sigma(y), \quad \sigma^+(x) = \bigcup_{i=1}^{+\infty} \sigma^i(x)$$
Subblobs$^+$ cycle free

\[ x \not\in \sigma^+(x) \]

The partitioning function $\pi$ associates equivalence relationship with $x$

\[ \pi : B \rightarrow 2^{B \times B} \]

Equivalence classes $\pi_i$ are orthogonal components of $x$

\[ \pi_1(x), \pi_2(x), \ldots, \pi_{k_x}(x) \]

$k_x = 1$ means a single orthogonal component (no partitioning)

Blobs in different orthogonal components of $x$ are disjoint

\[ \forall y, z \in \sigma(x) : \sigma^+(y) \cap \sigma^+(z) = \emptyset \]

unless in the same equivalence class
Simple Higraph

blobs

orthogonal components
Induced Orthogonal Components

\[ B = \{ A, B, C, D, E, F, C, G, H, I, J, K, L, M \} \]
\[ E = \{ (I, H), (B, J), (L, C) \} \]
\[ \rho(A) = \{ B, C, H, J \}, \rho(G) = \{ H, I \}, \rho(B) = \{ D, E \}, \rho(C) = \{ E, F \}, \]
\[ \rho(J) = \{ K, L, M \} \]
\[ \rho(D) = \rho(E) = \rho(F) = \rho(H) = \rho(I) = \rho(K) = \rho(L) = \rho(M) = \emptyset \]

\[ \pi(J) = \{ (K, K), (K, L), (L, L), (L, K), (M, M) \} \]

Induces equivalence classes \( \pi_1(J) = \{ K, L \} \) and \( \pi_2(J) = \{ M \}, \ldots \)
These are the orthogonal components
Higraph applications (add specific meaning)

1. E-R diagrams
2. data-flow diagrams (activity diagrams)
   edges represent (flow of) data
3. inheritance
4. Statecharts
Statecharts = state diagrams + depth + orthogonality + broadcast

- Reactive Systems (event driven, react to internal and external stimuli)
- like Petri Nets, CSP, CCS, sequence diagrams, ...
- graphical but formal and rigorous for
  - analysis
  - code generation
- solve FSA problems:
  - flat $\Rightarrow$ hierarchy $\Rightarrow$ re-use
  - represent large number of transitions concisely
  - represent large number of (product) states concisely
  - sequential $\Rightarrow$ concurrent
Depth (XOR), semantics through flattening
Orthogonality (AND), semantics through flattening
Broadcasting (output events)

Input Segment: nmnn
History States

Automonomous

Red

Green

Yellow

Manual

H

resume
to_manual

t
t
Stopwatch Example
Extensions

- **time**: `after(10s)`
- **guards**: `[OC in(C)]`
- **parametrized events**: `ev(p1,p2)`
- **narrowcast**: `destination.ev(p1,p2), destination->ev(p1,p2)`
- **states vs. variables**
- **arrow**: $R$, negative arrow: `not R`, absence of arrow: `don’t know`
- **don’t know blobs**
- **Zoom outs (interface)**