Petri Net Analysis (Conserved Properties)

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Overview

- Intro to conservation properties
- Steps to analyze conservation prop.
- Examples
- Problems/constraints with implementation
- Demo

Intro to Conservation Properties

- What is conservation?
  - Property to maintain a fixed number of tokens $\forall$ states reached in a sample path
- Why is it useful?
  - Sometimes tokens represent resources
- We will look at relaxed conservation
  - Not the whole Petri net satisfies conservation property

Conservation Definition

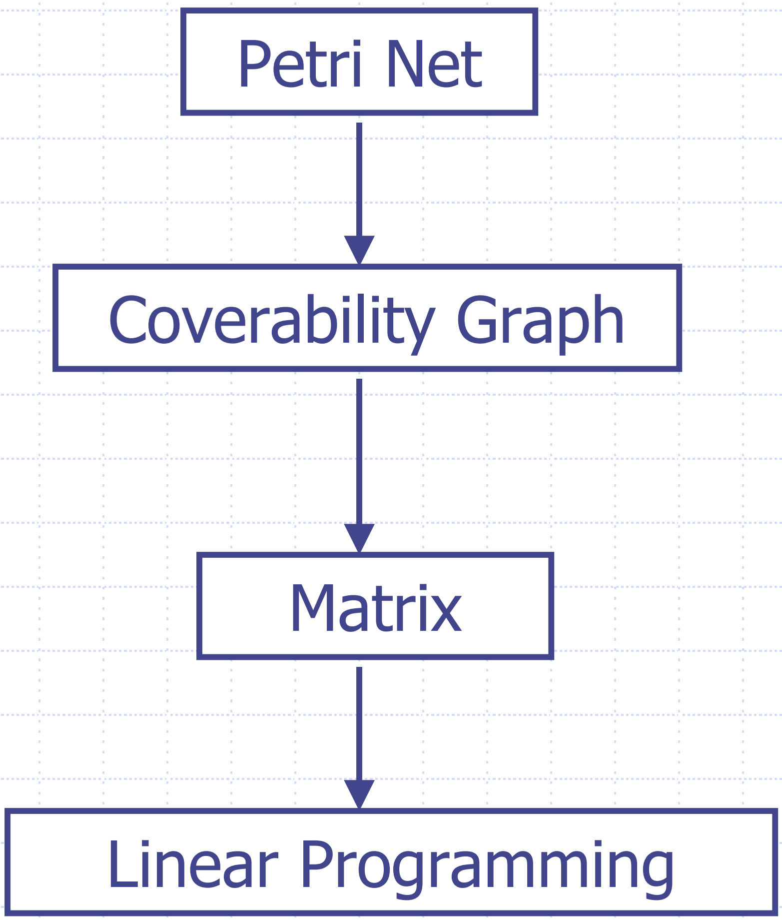
- Def'n: A Petri net with a given initial state $\mathbf{x}_0$ is said to be *conservative with respect to* $\gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]$ if

$$\sum_{i=1}^{n} \gamma_i x(p_i) = \text{constant}$$

For all states contained in all possible sample paths [1]

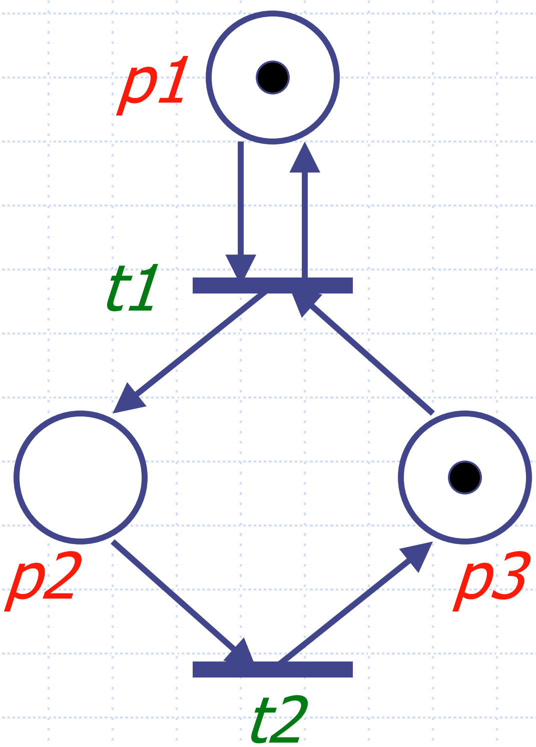
[1] Tadao Murata. Petri nets: Properties, analysis and applications. Proceedings of the IEEE, 77(4):541-580, April 1989

Steps to Analyze Cons. Prop.



Example

Step 1



Step 2

[p1, p2, p3]

[1, 0, 1]

t1 ↓ ↑ t2

[1, 1, 0]

Step 3

$$\gamma_1 + \gamma_3 = C$$

$$\gamma_1 + \gamma_2 = C$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$\gamma_1 \; \gamma_2 \; \gamma_3 \quad C$

Example (cont'd)

Step 4: reduce matrix using Gauss-Jordan elimination

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

Alternative: put C in first column

$$\left[\begin{array}{cccc|c} -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$C = \gamma_1 + \gamma_3$$

$$\gamma_2 = \gamma_3$$

Example (cont'd)

Step 5: solve linear programming problem using lp_solve [2]

$C = \gamma_1 + \gamma_3$

$\gamma_2 = \gamma_3$

2 independent vars:

γ_1, γ_3

lp_solve format:

min: $\gamma_1 + \gamma_3$

$\gamma_3 >= 0$

$\gamma_1 + \gamma_3 >= 1$

$\gamma_1 >= 0$

$\gamma_3 >= 0$

int γ_1, γ_3

[2] http://elib.zib.de/pub/Packages/mathprog/linprog/lp-solve/

Example (cont'd)

$\min: \gamma_1 + \gamma_3$

$\gamma_3 >= 0$

$\gamma_1 + \gamma_3 >= 1$

$\gamma_1 >= 0$

$\gamma_3 >= 0$

int γ_1, γ_3

$C = \gamma_1 + \gamma_3$

$\gamma_2 = \gamma_3$

Set $\gamma_1 = 1$: $\gamma_3 = 0$

$C = 1, \gamma_2 = 0$

Set $\gamma_3 = 1$: $\gamma_1 = 0$

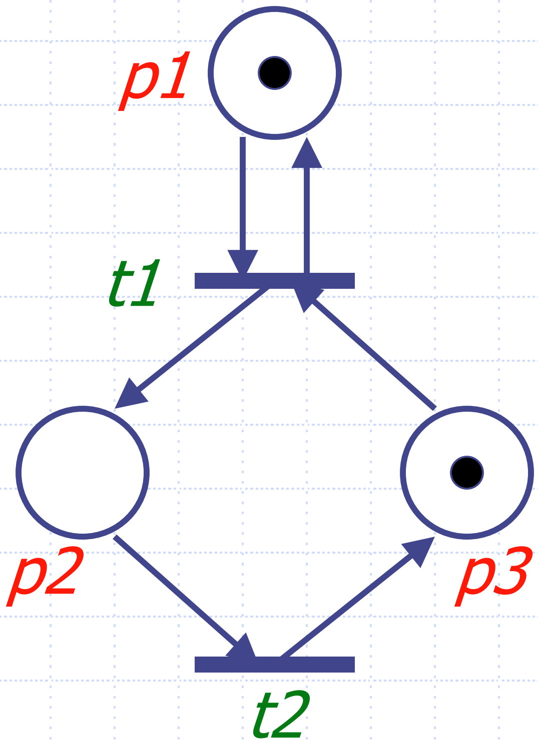
$C = 1, \gamma_2 = 1$

$$\sum_{i=1}^{n} \gamma_i x(p_i) = \text{constant}$$

x[p1] = 1

x[p2] + x[p3] = 1

Example (end)

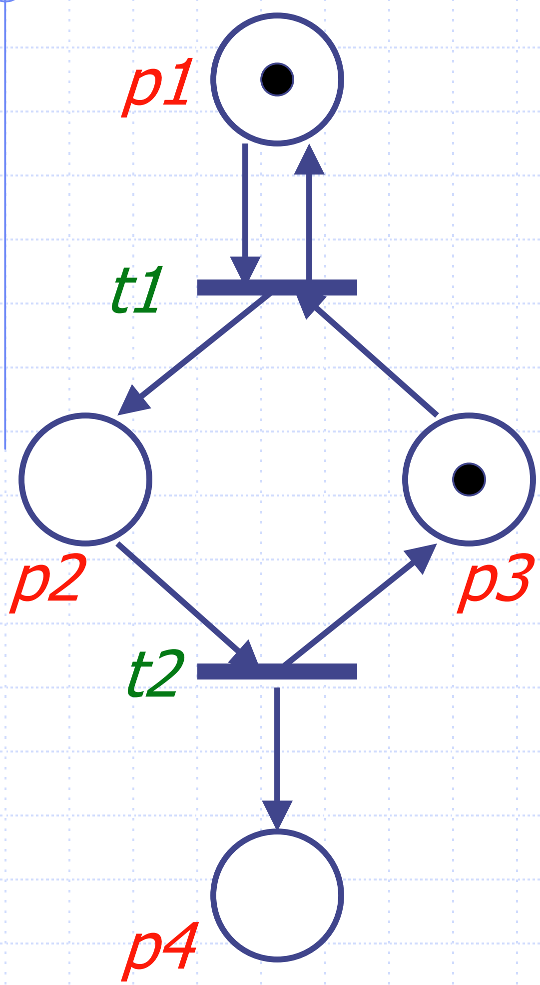


$$\sum_{i=1}^{n} \gamma_i x(p_i) = \text{constant}$$

$$x[p1] = 1$$

$$x[p2] + x[p3] = 1$$

Example with ∞ Capacity



[p1, p2, p3, p4]

[1, 0, 1, ω]

t1 ↓ ↑ t2

[1, 1, 0, ω]

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

γ_1 γ_2 γ_3 γ_4 C

$$\sum_{i=1}^{n} \gamma_i x(p_i) = \text{constant}$$

x[p1] = 1

x[p2] + x[p3] = 1

Problems/Constraints

- Not guaranteed to find solution
  - Integer linear programming problem
- Get less constraint equations than number of independent variables
  - Permute matrix columns & redo reduction
- Might not be unique solution

References

[1] Tadao Murata. Petri nets: Properties, analysis and applications. Proceedings of the IEEE, 77(4):541-580, April 1989

[2] lp_solve home page,

http://elib.zib.de/pub/Packages/mathprog/linprog/lp-solve/

[3] Petri Net conservation analysis assignment,

http://studwww.ugent.be/~dsooms/ftw/syssim/project2/

[4] Petri Net boundedness analysis assignment,

http://moncs.cs.mcgill.ca/people/hv/teaching/MS/assignments/assignment2/

Demo