Nuclear Magnetic Resonance Imaging

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Abstract

This paper gives an elementary introduction to NMR imaging, with emphasis on the electromagnetic content of the theory. The basic physical principles of NMR are presented, followed by the image encoding principles and an overview of the EM hardware used.

Contents

1 Introduction

Before Nuclear Magnetic Resonance (NMR), many scientists were taught that you cannot image an object smaller than the wavelength than the energy used to image it. It could explain why radio waves have been very lately thought to be used to image the inside of the human body, even if they are quite more inoffensive than the usual x-rays which can ionize atoms (it is worth noting that the only penetrating radiations in the human body are radio waves and lower frequencies; as well as x-rays and higher frequencies). The NMR phenomenon comes from resonance absorption of radiation from nuclear spins in an external magnetic field. The frequency of resonance is very well defined and depends on the magnetic field, thus a spatial encoding of a sample can be made by a spatial variation of the magnetic field and then measuring the different resonant frequencies observed for specific excitations. The usual scattering method to image an object is replaced by the magnetic field encoding and resonance signal.

We will start this paper with a brief overview of the physics of nuclear magnetic resonance, which is the basic phenomenon behind NMR. We will then describe how NMR can be used to image an object. We will finish with an overview of some of the EM hardware used in NMR.

2 Nuclear Magnetic Resonance

NMR was discovered independently by two groups of physicists headed by F. Bloch and E.M. Purcell in 1946 (see [2] for the historical references). It is fundamentally a quantum mechanical effect which arises when particles with a non-zero magnetic moment are put in a strong constant magnetic field superposed with a weaker orthogonal oscillating magnetic field with the correct frequency.

2.1 Basic Principles

We will consider here the spin-$\frac{1}{2}$ case, as it is the one used in NMR imaging. Let's say that we have a constant field $\mathbf{B_0}$ oriented in the z-direction. We know that the energy of a magnetic dipole μ in an external magnetic field $\mathbf{B_0}$ is given by:

$$H = -\mu \cdot \mathbf{B_0} \tag{1}$$

and the magnetic moment of a particle with spin $\mathbf{S}$ is:

$$\mu = \gamma \mathbf{S} \tag{2}$$

where γ is a constant called the *gyromagnetic ratio* for the particle. In the case where we apply a small oscillating magnetic field $\mathbf{B_1}$ in the x-direction, it is shown in [5, p.104] that the probability of transition from a spin up to a spin down state goes like a Lorentzian shape with resonant frequency $w_0 =$

$\mu B_0/\hbar$[1] and width characterized by the quotient B_1/B_0 (and so very narrow when $B_1 \ll B_0$[2]). This means that only radiation with a specific frequency $\nu_0 = w_0/2\pi$ (characterized by B_0 and called the *Larmor frequency*) can excite the spins of the particles. In NMR imaging, the particles considered are the single protons of the hydrogen atom ^{1}H, and those have a gyromagnetic ratio of $\gamma = 2.7 \times 10^8 s^{-1} T^{-1}$ and thus a resonant frequency of 42.6 MHz (radio frequency) for a field B_0 of 1 Tesla.

2.2 Larmor Precession

In quantum mechanics, the spin in the x-direction or the y-direction can be considered as a linear combination of spin up and spin down (in z-direction), and so the intermediate states in the spin up to spin down transition in a sample with lots of nuclei could be seen as the appearance of a transverse spin (and thus transverse magnetization). Instead of going in the quantum mechanic treatment of this phenomenon, we can simply look at the classical theory of electromagnetism which explains some of its features. Since the magnetic moment μ is related to the angular momentum (spin) $\mathbf{S}$ by equation (2) and that the torque $\mathbf{N}$ exerted on a magnetic dipole is given by $\mathbf{N} = \mu \times \mathbf{B}$ [3, p.174], then we have the classical motion equation for the dipole:

$$\frac{d\mu}{dt} = \gamma \mu \times \mathbf{B_0} \tag{3}$$

which has as solution, in the case of a static $\mathbf{B_0}$ field, that the dipole μ precesses about $\mathbf{B_0}$ on the surface of a cone at a Larmor frequency $w_0 = \gamma B_0$ (see figure 1(a)). In the case where we replace $\mathbf{B_0}$ by $\mathbf{k}B_0 + \mathbf{i}B_1 \cos wt$, then equation (3) expressed in a rotating coordinate system around the z-axis with angular frequency $-w$ becomes:

$$\frac{d\mu}{dt} = \gamma \mu \times \mathbf{k}(B_0 - w/\gamma) + \mathbf{i}B_{rf} \tag{4}$$

where $B_{rf} = B_1/2$ and is a constant in time, and the unit vectors are in the direction of the primed coordinate axis. By comparing this equation with equation (3), we see that the motion in the rotating frame is simply a precession around the *effective static* magnetic field $\mathbf{B_{eff}} = \mathbf{k}(B_0 - w/\gamma) + \mathbf{i}B_{rf}$ (see figure 1(b)). At magnetic resonance, $w = \gamma B_0$ and thus μ precesses around $\mathbf{i}B_{rf}$ with angular frequency $w_1 = \gamma B_{rf}$. This means that with the correct timing (which would only depend on B_{rf}), we could induce a rotation of 90^o to μ in the rotating coordinate frames and thus obtained a pure transverse dipole. If then the B_{rf} would be turned off, we would be left with a transverse dipole precessing around the z-axis with Larmor frequency w_0 (and this could induce an oscillating electric field in a coil with characteristic frequency which could be measured; this is the signal which is measured in NMR imaging).

[1] We note that μB_0 is simply the energy difference between the spin up and spin down state.

[2] B_0 is of the order of 1 Tesla in NMR imaging and B_1 is the order of 10 mT.

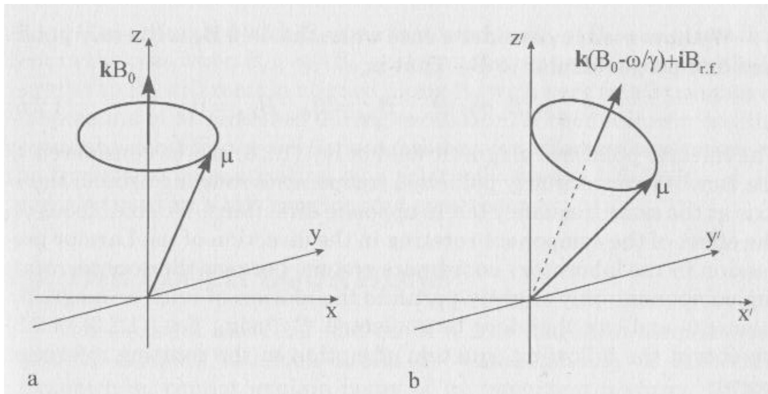

Figure 1: (a) Precession of the magnetic moment μ about $\mathbf{B_0}$ in the laboratory coordinate system; (b) precession of μ about the effective magnetic field $\mathbf{k}(B_0 - w/\gamma) + \mathbf{i}B_{rf}$ in the rotating coordinate system. Taken from [4, p.13].

2.3 Bloch model and relaxation time

We now need to make some macroscopic treatment of the process in order to link it with the real world. At thermal equilibrium, there will be more nuclei with spin up than spin down since they have lower energy in the magnetic field $\mathbf{B_0}$ (by equation 1). The probability of finding a nucleus in a particular spin state with energy E_m at thermal equilibrium is given by the Boltzmann statistics:

$$P_m \propto e^{-E_m/k_B T} \tag{5}$$

where k_B is the Boltzmann constant and T is the temperature. The difference in population of the different energy levels (spins) is small (e.g. $(n_+ - n_-)/n \simeq 10^{-5}$ at room temperature and at 1 Tesla); but the large number of hydrogen protons in a human sample yields a macroscopic difference between the number of spin up and down and thus a macroscopic magnetization $\mathbf{M} = \rho\langle\mu\rangle$ where ρ and $\langle\mu\rangle$ are the concentration and average magnetic moment of the nuclei, respectively. According to the results of the preceding section, we could rotate the magnetization vector by 90^o with a r.f. radiation pulse at the Larmor frequency w_0 with the magnetic polarization in the x-y plane and with the correct duration determined by B_{rf}. After the pulse, the average magnetization would keep precessing around the z-axis at the Larmor frequency w_0, and according to Faraday's law, it would induce an alternating current with the same frequency in a coil with a normal in the x-y plane. But according to the Bloch model (see [4, p.15]), the transverse magnetization decays exponentially to 0 with a characteristic time of T_2[3] known as *spin-spin relaxation time.* The measured signal is thus a decaying oscillating current, called the *Free Induction*

[3]This parameter depends on the environment in which the hydrogen atoms are and thus can be used to differentiate different type of tissues in NMR imaging.

Current (FID), which will be proportional to ρ. The decay is due to small interactions between the microscopic dipoles, which makes small perturbation amongst the Larmor frequencies of the dipoles and cause them to *dephase* and average to zero after a certain time. Also, the magnetization in the z-direction gradually relaxes to its initial value due to thermal perturbations and internuclear interactions. In the Bloch model, the relaxation is also given as an exponential law:

$$M_z = M_0(1 - e^{-t/T_1}) \tag{6}$$

where T_1 is called the *spin-lattice relaxation time*. This sets the scale of time after which we can repeat a r.f. pulse to excite again the spins.

3 NMR Imaging

We are now ready to understand how we can do NMR imaging. A simplified version of it could be described with the following steps:

1. The patient is put in a strong uniform magnetic field $\mathbf{B_0}$.
2. RF coils are used to select a slice of the sample to map (see section 3.1) by rotating the nuclear moments by 90^o only in a specific slice.
3. Gradient coils are used to create a precise linear spatial variation of the magnetic field to encode the position of the excitation in the slice (see section 3.2).
4. RF resonant coils measure the FID produced by the decaying precession of the nuclear moment in the spatially encoded magnetic field.
5. The signal can be inverse Fourier transformed to obtain a spatial distribution of the hydrogen nuclei density.

3.1 Slice selection

To select a thin slice of the sample to map, a small magnetic field gradient in the z-direction is superposed (with component parallel to the main field) onto the main uniform field. This means that B_0 will be constant only in x-y planes and thus that the nuclei have constant Larmor frequency planes by planes. Since we saw in section 2.1 that the width of the magnetic resonance effect was quite narrow when the exciting field is very small compared to the constant field, only a specific slice will be excited if the exciting radiation has only one narrow band of frequencies. The time variation of the B_{rf} field usually used (as for equation (4)) is a sinusoidal with frequency w_0 multiplied by a sinc-shaped pulse (shaped like $\sin x/x$). This signal is used because it is a pulse with a narrow square frequency distribution (as can be seen from its Fourier transform), and thus it will only excite a narrow slice. The r.f. pulse is made so that it has just the right duration to rotate the spins by 90^o.

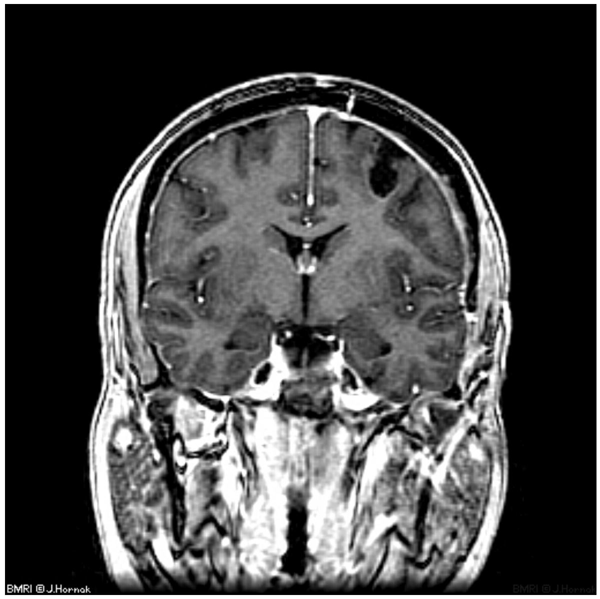

Figure 2: **Axial head NMR image with pathology.** Can you see the pathology? Image taken from [2].

3.2 Gradient Encoding and Fourier Transform

Just before recording the FID of the precessing spins, gradient coils are turned on to encode the x-y position of the spin volumes. A *frequency* of precession encoding is made by adding a small steady linear variation in the z-field along the x direction (the frequency of precession is determined by the magnetic field, as seen in section 2.2). Also, a *phase encoding* is made by adding for a specific interval of time a small linear variation in the z-field along the y direction (the difference of frequency along the y direction will cause a phase shift between the different dipole precessions, which will come back to the same frequency when the y-gradient will be turned off). After those two quick encoding, the FID signal is recorded using r.f. resonant coils. Because of the phase and frequency spatial encoding, the density distribution of the nuclei in a slice can be found back by invert Fourier transforming the recorded current signal. The density of nuclei can then be depicted on a gray-scale image (an example of typical NMR image is given in figure 2).

Of course, this rapid turning on and off of gradient coils could induce eddy currents in the conducting structures which in turn induce both temporally and spatially varying magnetic field which could create distortions in the image (which will be one kind of *image artifact*) and attenuation of the signal. Advanced techniques try to reduce those effects (as well as the inherent inhomogeneities of the field by specially designed gradient waveforms which

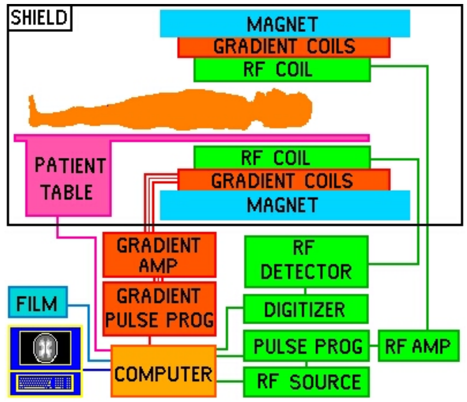

Figure 3: **Major components.** Image taken from [2].

compensate for the created eddy currents – see [4] and [2] for more information about image artifacts.

4 The Hardware

We end this paper with a brief presentation of some of the EM hardware used in NMR. Figure 3 gives more details about the general setting.

4.1 The Magnet

It is important to have a strong uniform main magnetic field in order to have a stable magnetic resonance effect and also because the position of the nuclei is encoded with the field, so that any undesired variation in the field causes geometric distortions of the image (variation in the field can also be caused by metallic objects on (or in!) the patient). The typical strength of the fields used in NMR imaging is between 0.5 and 2 Tesla [1]. Such strong fields cannot be obtained with resistive magnets (coils with electric current), so superconducting magnets are used instead. The magnet is bathed in liquid helium to keep it in the superconducting regime.

4.2 RF Coils

The main functions of r.f. coils are to excite the magnetization in a sample and to receive the signal produced by the excited magnetization (they can sometimes act both as a receiver and as a transmitter). They are simply coils which are in series with an inductor and a capacitor in order to have a resonance behaviour with angular frequency

$$w = 1/\sqrt{LC} \tag{7}$$

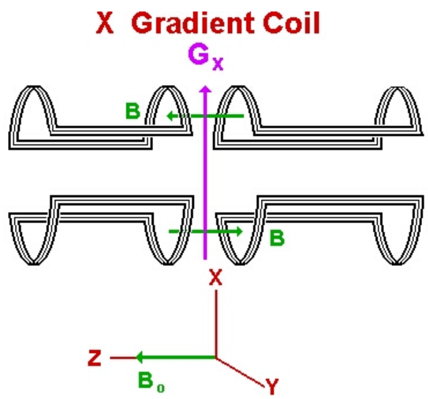

Figure 4: **Galey coils to produce gradient in the transverse direction.** Image taken from [2].

where L is the inductance and C, the capacitance. They are sometimes shaped according to specific parts of the body of the patient (leg, wrist, head, etc.) in order to have a closer measurement of the signal.

4.3 Gradient Coils

The field gradient in the z-direction is simply made with a normal cylindrical coil. The field gradient in the x and y-directions can be made using *Galey coils* as is shown in figure 4.

References

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