The transport of charges investigated by Hall Effect

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Abstract

Voltages across a germanium crystal are recorded on a range of temperatures going from 150 to 383 K. Impurities, temperature and the presence of a magnetic field (causing the Hall effect and magnetoresistance) modify the mechanism of conduction. Measurements of the temperature dependence of the resistivity ρ together with the Hall coefficient R_H give information on several properties of the semiconductor. The n-type nature of the dopant has been successfully established and its concentration, approached by a factor of two. The bandgap energy E_g measured was found in the right range of expected values. Finally, two types of charge carriers' mobilities were discriminated and the power law of $\rho(T)$ in the intrinsic region was verified, as well as the power law for $\rho(B)$ (magnetoresistance).

1 Introduction

The 20th century has been the theater of deep transformations all domains: politics, arts, science,... The revolution in the communications and computers will surely become its principal legacy. This was initiated by the discovery of the transistor and its technologic development since 1947. These devices are based on junctions of semiconductors into which a current can flow in a unique direction. The physics behind such peculiarity is a direct consequence of the Quantum Mechanics. The properties of the semiconductors revealing a wealth of possibilities, a new branch was born: the Solid State Physics.

However, the first insights on the theory of conduction into materials came from Edwin Hall who found in 1879 an effect which has his name. He remarked that a voltage appears across crystal's sides when it is inserted into a magnetic field and a current passes through it. A successful explanation was given by considering the deflection of the charge carriers due to the magnetic force. Today, the Hall Effect combined with the knowledge of the resistivity of a semiconductor can be used to deduce many of its properties: the process of intrinsic and extrinsic conduction, the nature of the dopant, the concentration of impurity carriers, the conduction mechanism, the bandgap and the mobility of the carriers. Using a germanium crystal, the aim of this experiment is to illustrate how such measurements yield to all this information.

2 Theory

2.1 Band structure in crystals

The first property describing a material is its ability to transport charges under the influence of an applied electric field \vec{E} . This situation is governed by Ohm's Law (see [1]):

$$\vec{J} = \sigma \vec{E} \tag{1}$$

where \vec{J} is the current density and σ , the conductivity. The latter changes dramatically according to the nature of the material: the good conductivity of a metal can be greater than insulator's one by a factor of 10^{24} (see [2]). A successful explanation for such enormous variations has been achieved by the band theory of solids. The energy bands arise directly when one solves the Schrödinger equation for electrons subject to a periodic potential as a model for the structure of a crystal (see [3]). Another approach depicted in [4] considers isolated atoms which become neighbors like in a solid. When two of them are approached, their wavefunctions overlap and this produces a splitting in the energy levels as in figure 1a. In presence of more neighboring atoms, new degeneracy states appear. For a lattice containing a large number of atoms, this phenomenon results in a continuum of allowed states for the electrons called a band (see figure 1c). Note then that a gap separates successive bands by forming a region of forbidden energies for the electrons.



Figure 1: Splitting of the energy levels due to atomic lattice. [2, p.435]

We shall see that the occupation and the position of the two highest bands completely determines the conductivity of a solid. The valence band is the outermost containing electrons while the lowest one which possesses available states is called the conduction band. The separation between the two corresponds to the energy gap E_g of the material. As shown in figure 2, this value indicates the nature of conduction. In metals, no gap exists and electrons can move freely because the unoccupied states just above are attained with a small amount of additional energy. On the other hand, insulators possess large energy gaps (~ 10 eV) compared to k_BT at room temperature (0.025 eV), where k_B is the Boltzmann constant. The Fermi-Dirac distribution predicts that at normal temperatures very few electrons can be thermally excited to the upper band, resulting in a low conductivity. The semi-conductors lie between those two extremes and thus have very interesting properties.

2.2 Semiconductors and impurities

The energy gap for semiconductors is of the order of 1 eV. At low temperatures, they remain poor conductors as no electrons populates the conduction band. However, as the temperature increases, the thermal excitation through the narrow gap becomes more probable. The conductivity is thus expected to increase at sufficiently hight temperature.

A particularity for semiconductors is that both positive and negative charge carriers are present. When an electron is promoted from the valence band to the conduction band (see figure 3), it leaves behind a vacant crystal site, so-called hole. This electron-deficient site in the otherwise filled valence band acts like a positive charge: a nearby valence electron transfers to this empty site, leaving a hole behind, at its original place. This process is repeated and the hole



Figure 2: Energy band diagrams for different kinds of solids.[2, p.437-438]a) conductor (metal)b) insulatorc) semi-conductor



Figure 3: Flow of charge carriers in a pure semi-conductor excited by an electric field. [2, p.439]



Figure 4: Energy band diagrams for two kinds of doping of a semiconductor. [2, p.443-444] a) p-typed doping b) n-typed doping

migrates through the valence band. In pure crystals, the number n of negative charge carriers has to equal p, the number of positive ones. The transport of this pair of charges created by thermal excitation in a semiconductors has a name: *intrinsic conduction*. With a high energy approximation (i.e. $E \gg k_B T$) (see [5]), n is described by a Boltzmann distribution:

$$n \simeq \left(\frac{2\pi m_e k_B T}{h^2}\right)^{\frac{3}{2}} \exp{-E_g/2k_B T}$$
(2)

All important semiconductors are tailored to the needs of a particular device by a process called doping. It consists of an addition of specific impurities to the structure of the crystal. According to their effect, we subdivide them in two categories. A *donor* is an impurity lying at an energy E_d just below the conduction band and capable of donating electrons to it (see figure 4a). Since the charge carriers are negative electrons, the semiconductor is known as an n-type. Alternatively, we can insert *acceptor* atoms as in figure 4 which accept electrons from the valence band just below. Now the charge is carried by positive holes, giving a p-type crystal. The conduction due to any of these impurities is called *extrinsic* and as a first consequence, the symmetry in the number of charge carriers is lost. i.e. $n \neq p$. Making a low temperature approximation, the number of electrons in the conduction band is given by

$$n = N_d \left(\frac{2\pi m_e k_B T}{h^2}\right)^{\frac{3}{2}} \exp{-E_d/2k_B T}$$
(3)

where N_d corresponds to the number of donors. In the case of the germanium,

a separation E_d of 0.01eV has a relatively small k_BT energy associated with it in comparison to an E_g of 0.67eV [5, p.82]. Hence, for temperatures $T \gtrsim 120 K$, the number of impurity carriers in the conduction band becomes saturated ($n \simeq N_d$). As the temperature increases, these act like a constant number of free electrons and there is a gradual increase of the density of intrinsic carriers [5, p.82-83].

2.3 Resistivity

The types of the charge carriers and conduction have been exposed. The next step considers the quantitative temperature dependence of the transport of charges. For this purpose, we rather use the dual quantity ρ , the resistivity of the material, which is defined as the inverse of the conductivity. This macroscopic property is linked to the motion of the charges in the crystal. When no field is applied, an electron executes a free path between collisions, moving in a random fashion as a molecule in a gas. Under the influence of a field \vec{E} , it drifts in the direction given by the field. The drift velocity v_d is related to the field's strength by:

$$v_d = \mu \|\vec{E}\| \tag{4}$$

where the proportionality constant μ is called the mobility. The current density restricted to one dimension can be expressed as a function of the velocity of the charges by:

$$J = env_d \tag{5}$$

e being the fundamental charge. Combining Equations 1 to 4, we obtain this relation for the resistivity if one type of carriers is present in the semi-conductor:

$$\rho(T) = \frac{1}{\sigma} \propto \frac{1}{\mu} T^{\frac{-3}{2}} \exp \frac{E_g}{2k_B T}$$
(6)

In the extrinsic region, the density of carriers remains almost constant and a power law is expected [5, p.92]:

$$\rho_{ext}(T) \propto T^{\beta} \tag{7}$$

where β is a constant. Combining (4), (5) and (7), the behaviour of the drift mobility μ_d linked to the drift velocity and the resistivity goes like:

$$\mu_d \propto \frac{1}{\rho_{ext}} \propto T^{-\beta} \tag{8}$$

Furthermore, considering some approximations about the thermal velocity of the charge carriers and their mean free path, we can derive as in [5] a simple approximation for the temperature dependence of the resistivity and we get $\beta = 3/2$ for the region of constant n:

$$\mu_d \propto T^{\frac{-3}{2}} \tag{9}$$

For larger T, in the intrinsic dominated region, the behaviour of (6) becomes dominated by the exponential:

$$\rho_{int}(T) \propto \exp \frac{E_g}{2k_B T} \tag{10}$$

On the other hand, the resistivity can also be measured directly in terms of the resistance R of the whole sample and its geometry:

$$R = \frac{\rho\ell}{A} \tag{11}$$

where A is the cross-section area and ℓ , the length of the sample parallel to the current. Applying again Ohm's Law for an entering current I and a potential difference V produced in the longitudinal direction, we get

$$R = \frac{V}{I} \tag{12}$$

and finally ρ as a function of measurable parameters:

$$\rho = \frac{AV}{I\ell} \tag{13}$$

2.4 Hall Effect

If the resistivity yields information about the bandgap, the mobility of the carriers and the conduction's behaviour, other basic properties of the semiconductors can be investigated using the Hall Effect. The measurement of the Hall coefficient R_H will reveal the nature of the dopant, the concentration of impurity carriers, the conduction mechanisms and the distinction between the two types of carriers. This effect can be depicted as on figure 5: a current I flows in the x direction through a crystal placed into a constant magnetic field B parallel to the z axis. Under the influence of the latter, the moving charges deflect on one side, creating a lateral difference of potential V_H . Assuming that all carriers share the same drift velocity, we provide two derivations of R_H , according to the number of carriers.

2.4.1 One type of carrier

This section describes the Hall Effect when the carriers share the same charge, as in an n-type semiconductor. At equilibrium, the magnetic force is compensated by the repulsion of the accumulated charges, i.e. by the Hall field E_H along the y dimension:

$$F_m = e(\vec{v}_x \vec{B}) = ev_d B = F_H = eE_H \tag{14}$$

noting that the nature of the charge carrier doesn't alter this result which is invariant of a simultaneous sign reversal of both the velocity and the charge.



Figure 5: Hall effect

We can now define the Hall coefficient (see [6]) as the ratio and find this simple result:

$$|R_H| \equiv \frac{E_H}{J_x B} = \frac{v_d}{J_x} = \frac{\mu E_x}{J_x} = \frac{\mu}{\sigma} = \frac{1}{ne}$$
(15)

using equations (14), (4) and (5) in the intermediary steps. The R_H 's sign depends directly on the sign of the charges and becomes negative when electrons are considered. Also, this quantity is inversely proportional to the number of charges carriers n, allowing an estimate of the concentration of impurity carriers in the extrinsic region. From (15), we define the mobility found in terms of R_H , namely the Hall mobility μ_H :

$$\mu_H \equiv R_H \sigma = \frac{R_H}{\rho} \tag{16}$$

where A is the cross-sectional area of the crystal's face where a constant current I enters, flowing in the ℓ direction. Also, the Hall angle ϕ gives an idea of the effect's strength by a ratio of V_H and the voltage V_x producing the current. It can be related experimentally to μ_H :

$$\phi = \frac{V_y}{V_x} = \frac{tE_H}{\ell E_x} = \frac{\mu_H B w}{\ell} \tag{17}$$

and then to R_H by (16).

2.4.2 Two types of carriers

In this case, we generalize part 2.4.1 by taking into account the p carriers of opposite charge which have a Hall coefficient with a different sign for the same electric field E_x . The full derivation proposed in [5, pp.86,87] gives a general formula for R_H :

$$R_{H} = \frac{\mu_{h}^{2} p - \mu_{e}^{2} n}{e(\mu_{h} p + \mu_{e} n)^{2}}$$
(18)

where μ_e and μ_h are the mobilities of the electrons and the holes respectively. Since $\mu_e > \mu_h$ in general, the R_H which still depends on T can see its sign change provided that p > n. In other words, the "Hall coefficient inversion" is typical of p-types semiconductors.

2.5 Magnetoresistance

The presence of a magnetic field as in the Hall Effect will also affect the resistivity of the sample. The phenomenon is known as magnetoresistance and is caused by the curved trajectory that the moving charges follow according to the Lorentz force. By assuming that the modulus of the speed of the charges is kept to v_d , the additional travelling distance relative to the straight line is calculated. This deviation reduces the effective x-component of the current and hence increases the resistance. Keeping the first terms in a perturbation expansion leads to a quadratic correction of the resistivity by the magnetic field B:

$$\rho(B) = \rho(0) + (\text{const.})B^2 \tag{19}$$

which agrees with [6, p.25].

3 Experimental Procedure

3.1 Setup and Apparatus

As suggested by the theory, the analysis of the properties of a semiconductor required a temperature control of the sample, a voltage measurement device and an efficient data acquisition system.

The first task was achieved by a thermal container where we slid a protecting copper cylinder, closed at the bottom and sealed at the top (see figure 6). The latter contained the Germanium sample in its superior part which was surrounded by an electric heater receiving DC current from a voltage controlled power supply. The lower part of the so-called Hall probe could be cooled by filling the thermos with liquid nitrogen. Since the temperature was not distributed uniformly among the components, the thermocouple was directly connected to the germanium crystal.

A closer look at the sample as in the block diagram of figure 6 reveals six contacts measuring potentials. The voltage at point 1 is directed to a cold junction compensator which references this reading to 0° C. This forms a thermocouple and the resulting voltage can be converted into temperature according to tables given in the lab manual. This signal and the other sites are all connected to a digital multimeter (Keithley DMM). In a cycle of 20 seconds, it read these voltages and recorded seven potential differences $(V_1 \text{ to } V_7)$ and V8, the potential representing temperature. The later acquisition process was started by the hall program on the computer. The data was sent to it via a GPIB card and stored in files of 9 columns: the eight voltages and a computer's time recording in the ninth column. At the end of each cycle, the new values were exposed on the screen as in figure 7. Voltages V_5 and V_6 were meaningful for the resistivity part of the experiment while V_1 and V_2 were two equivalent readings of the Hall voltage used in 3.4. Other voltages were there for diagnostic purposes. In order to create the effects that we wanted to observe, a current source (referring back to figure 6) provided a constant¹ current of 1 mA passing through the far ends of the crystal. Also, for the measurement of the Hall coefficient, the Hall probe was introduced inside a uniform² magnetic field. An Alpha Scientific electromagnet, powered by its own DC power supply, produced the magnetic field in a confined region of space and had to be water cooled to prevent overheating. A Lakeshore 412 Gaussmeter was used to determine the magnetic field's intensity.

¹The current was confirmed to be constant as the reading V_4 remained the same during the whole run (±0.001mV).

 $^{^2 \}rm Was$ found to be uniform at $\pm 0.007 \rm ~T$ during the experiment.



Figure 6: Experimental Setup.



Figure 7: Voltage measurements. This image was taken from the lab manual. The same figure was shown on the computer screen, with the measured values in the corresponding circles. The grey rectangle represents the Germanium sample. V5 and V6 were used for the resistivity measurements; V1 and V2 were used for the hall voltage measurements. There was also a V8 output taken from the thermocouple for temperature measurement (not shown on figure).



Figure 8: Crystal dimensions. This image was taken from the lab manual. The dimensions w1, w2, L5 and L6 are associated with the voltages V1, V2, V5 and V6 respectively.

Finally, the useful dimensions of our Germanium sample are given in table 1. The lengths were measured with a travelling microscope.

length, L5	$4.9\pm0.4~\mathrm{mm}$
length, L6	$4.9\pm0.4~\mathrm{mm}$
width, w1	$1.97\pm0.05~\mathrm{mm}$
width, w2	$1.91\pm0.05~\mathrm{mm}$
thickness, t	$1.05\pm0.02~\mathrm{mm}$

Table 1: **Dimensions of Germanium crystal.** The meaning of the letters is given in figure 8. The high incertitude on the measured values with the travelling microscope comes from the difficulty to maintain sample setup in place for the measurement (the sample would move between two endpoint measures).

3.2 Measurement of the resistivity as a function of temperature

Having information about the properties of the germanium [5], the temperature's range was maximized in order to cover well both regions of conduction (in the limits of our setup). At the beginning, the sample was cooled as much as possible: 150 K was the minimum which has been attained. We started the hall program when the temperature started increasing (since it usually decreased too

Temperature Evolution



Figure 9: **Temperature Evolution** This shows how the temperature varied during the resistivity part of the experiment. When a line stops, it means the program had crashed. The circled region corresponds to a quick increase of temperature that we never succeeded to avoid, due to the end of the evaporation of the liquid Nitrogen. The runs for the Hall effect part of the experiment are similar.

fast too take meaningful results). Ideally, all the measures should be done in a thermal equilibrium. From the minimum to about 0°C, this objective was approached by keeping the uncontrolled rate of increase as low as possible (but keeping in consideration the finite amount of time which was available for this experiment). Referring to the time evolution graph of our run (see figure 9), we can notice that in this range, the heating rate didn't exceed 0.3 K per cycle of 20 seconds, except in a region of sudden acceleration (shown in a circle in figure 9). This will be explained in the data analysis part, section 4.1.1. We tried to attenuate this effect by adding some insulation. For temperatures above room temperature, the same low rate (0.3 K/cycle) was targeted by adjusting carefully the voltage of the heater. We terminated the run at a maximum of 383 K, since for higher temperatures, there was a significant danger that the solders on the sample would meld. One can also notice on figre 9 that the time was reset often to 0. This indicates that the acquisition program had crashed, due to some bug in the configuration of the input device on DOS. This happened regularly and rendered the data acquisition tricky. When those crashes happened, we simply restarted the program with a different output file name.

3.3 Measurement of the Hall Effect as a function of temperature

Essentially, the same strategy than the previous one was used to control the temperature. The only difference concerns the maximum attained, lowered to 375 K in reason of the water cooling of the magnet. The magnetic field was initially set to 4.98 kG. A measure at the end indicated a small decrease of 0.05 kG. The measurement had to be done twice. The first time, the sample was placed perpendicular to the constant magnetic field. This was determined by maximizing the Hall voltage V_1 , given directly by the digital multimeter. The second run was taken with the probe turned by an angle of 180 degrees. The pairs of contacts being not exactly face to face, this procedure allowed us to eliminate the offset voltage created by misalignment of the solders and the longitudinal current.

3.4 Measurement the magnetoresistance as a function of the magnetic field intensity

The Lakeshore's sensor and the probe fixed in the electromagnet, we have studied the influence (at fixed temperature) of the magnetic field's strength on the resistivity. In order to reduce the variations of the magnetic field due to the heating of the magnet, the readings were executed on a short period of time. For example, in less than three minutes, the voltage governing the magnetic field had been increased 22 times and the range covered went from 0.51 kG, the residual field, to 7.88 kG, the maximum allowed by the source. A reading outside the magnetic field was also done in order to determine the resistivity without magnetoresistance. At each step, the intensity on the gaussmeter and the corresponding stabilized voltage V_5 were recorded.

4 Data and Analysis

4.1 Resistivity

4.1.1 Processing the data

All the graphs and data analysis were done using Micro\$oft Excel, for its easiness of data manipulation.

The raw data recorded by the hall program on the DOS workstation gave the seven voltages shown on figure 7, and the voltage of the thermocouple. To translate the thermocouple voltage automatically to temperature, we used the hall_process program³. This program created eight files: one for each voltage recorded on the computer. One file gave the temperature in function of time (figure 9 was created using this file). The other ones gave the voltage in function of temperature bins of a certain size (1K by default). The average voltage per bin, its standard deviation and the number of points falling in the

³Programmed in C by Mark Orchard-Webb.

bin were recorded in different columns in those files, in a format easily analyzable with other programs such as gnuplot. The program excluded by default the temperature bins which had less than two points (one or zero). This resulted in gaps in the voltage vs. T curves where the rate of change of temperature was too fast (more than 2 degree/minute). As we can see in figure 9, this happened for the resistance part of the experiment in the temperature region between 165 K and 195 K (the circled region). The same sharp increase behaviour also occurred for the Hall effect part. This could be explained by a phase transition behaviour: the slow increase of temperature around 150 K could be due by the evaporation of the Nitrogen, which would use the ambient thermal energy, preventing a too quick increase of the temperature of the sample. Once all the Nitrogen is evaporated (which seemed to happen when the sample was around 165 K), this heat sink would no more be present to temperate the heat flow between the outside world and the sample in the "isolated" chamber. Since heat flow is proportional to the gradient of temperature, the rate of heating was higher at first, as we can see in figure 9.

In order to prevent data gaps in this region, we used a bin size of 3 K instead of 1 K. Figure 10 shows the temperature dependance of V5 and V6, as given by the program hall_process. First, we can see on this graph the difference between a 1 K binning (shown in red) and a 3 K binning for V5 (shown in blue diamonds). The arrow points to the data gap for the 1 K bin V5 curve, caused, as explained earlier, by a too fast increase of temperature. The 3 K binning V5 curve is seen to fill this gap. The other gaps which we can see on this graph (around 208 K and 236 K, for example) weren't caused by the rate of change of temperature (since the gap is still present for higher bin size), but is due to the crash of the Hall program while taking measurements. Indeed, by looking at the temperature evolution graph in figure 9, we can see that those temperatures correspond exactly to the temperatures where the program had crashed (inferred by the end of a temperature curve). We'll come back to these two aspects when explaining the results in the intrinsic and extrinsic conduction regions.

4.1.2 Resistivity calculations

V5 and V6 give the voltage in the direction of the current in the sample, so we can use them to compute the resistivity with equation (13), given in section 2.3. The area A was computed using the average of the widths w1 and w2 shown in figure 8

$$A = \frac{(w1 + w2)t}{2}$$
(20)

where we use the same symbols as in figure 8 and table 1. We always used a current of 1.00 ± 0.01 mA in this experiment, so for the remaining of the analysis, we can write:

$$\rho = kV \tag{21}$$

where k is a constant given in table 2.



Figure 10: V5-V6 voltage, $\mathbf{B} = \mathbf{0}$. The points in red represent the V5 voltage in function of temperature with bin size of 1 K, whereas the blue diamonds are with bin size of 3 K. We can notice the data gap in the 165 K to 195 K region in the 1 K case, which is filled in the 3 K case. The points in green represent the V6 voltage. The errors (given by the standard deviation in the bins) are plotted only for the V6, for clarity of the graph, and also because the errors are very similar for the two other curves. The errors for V6 varied between 0.1% and 0.8%, with the maximum error reached in the 360 K region (where the rate of change of V6 vs. T is high), and in the 180 K region (where the rate of change of temperature vs. time was high).



Table 2: Proportionality constant for ρ

12.0 10.0 ♦ using V5 ♦ using V6 8.0 Resistivity (0hm * cm) 6.0 4.0 2.0 0.0 100 150 200 250 300 350 400 Temp (K)

Resistivity when B = 0

Figure 11: Resistivity in function of temperature when $\mathbf{B} = \mathbf{0}$. The errors bar on ρ computed with V5 include the 10% systematic error arising from k of table 2, and the random error arising when measuring V5 (around 0.5%).

The V in equation (21) can be either V5 or V6 since we measured the same value of L5 and L6 on our sample. The 10% error on k will contribute a systematic error throughout this analysis, but doesn't affect the behaviour of the resistivity curve as the random error on V will. The resulting resistivity curves in function of temperature is shown in figure 11. The (total) error for the resistivity using V5 is also plotted, and we can thus see that the points computed with V6 are reachable within the errors bars of ρ computed with V5, giving a sanity check for our measurements.

The table 3 give a comparison of our measured resistivity at 300 K and the one given in the specification of the samples. We can see that we are roughly in a factor of two of the specifications. As we were told by the lab guru Fritz Buchinger, this is usual in this kind of experiment.

Several interesting properties can be inferred from figure 11. First of all, we can see that the resistivity reaches a maximum around 310 K. In this region, there is a transition between the dominance of the extrinsic charge carriers and

measured resistivity at 300 K	9.1 ± 0.1 ohm*cm
resistivity given in specifications of the sample	$3.8 \text{ ohm}^* \text{cm}$

Table 3: Measured resistivity at 300 K.

the dominance of the intrinsic charge carriers.

The increase of the resistivity in the 150 K to 300 K region can be explained as follows. The density of extrinsic charge carriers was already saturated after 120 K, as mentioned in the theory, section 2.2. On the other hand, the thermal energy in this region is not enough to obtain a significant number of intrinsic charge carriers. The thermal energy given when increasing the temperature in this region then only increases the number of collisions of the electrons in the crystal. In consequence, the drift velocity of the charge carriers decreases, and thus the resistivity increases.

After the 310 K turning point, the thermal energy is starting to be sufficient to excite a considerable amount of intrinsic charge carriers. By witnessing a decrease of resistivity in this region, we conclude that the increase of the number of intrinsic charge carriers available dominate the effect of the diminution of their mobility (due to thermal energy increase). It thus seems logical to analyze the two regions separately.

4.1.3 Intrinsic conduction region

From our previous analysis of the figure 11, we expect the intrinsic conduction effect to becomes significant when the resistivity is decreasing, i.e. after 320 K. Taking the natural logarithm of both sides of equation (10), we have

$$\ln \rho_{int} = \frac{E_g}{2k_B T} + \ln C \tag{22}$$

where C was the proportionality constant. A plot of $\ln \rho$ vs. 1/T in the intrinsic region is shown in figure 12. A linear relationship is found roughly after 350 K, indicating that equation (22) is a valid approximation only after 350 K. A linear fit of the points with a red box is shown on the graph. The other points are seen to depart from the line, so were excluded. From the slope of the line, we could deduce the energy gap E_g , and compare it with the expected value. The results are shown in table 4. As we found in [7], E_g for the germanium depends in reality with temperature with the relationship:

$$E_g = 0.742 - 4.8 \times 10^{-4} \cdot \frac{T^2}{T + 235}$$
 (eV) (23)

But still, we see from table 4 that the approximations made in deriving (22) yield an E_g which is in the right energy range. We can also mention that considering V5 or V6 separately didn't change the results.



Resistivity in Intrinsic Conduction region

Figure 12: **Resistivity in intrinsic conduction region.** The resistivity used for this plot was the average one computed from V5 and V6. The x-axis was inverted so that temperature increases to the right, for correspondence with figure 11. No errors are shown on the plot since the systematic error is irrelevant for the relationship studied (it is absorbed in the C of equation (22)), and the random error is too small to be shown. A linear fit was done using Excel for the points marked in a red box. The temperature range was from 320 K to 380 K, with the fit done for points above 350 K.

measured E_g (with ρ from 350 K to 380 K)	$0.648 \pm 0.006~{\rm eV}$
using V5 only	$0.649\pm0.006~\mathrm{eV}$
using V6 only	$0.648\pm0.006~\mathrm{eV}$
expected E_g for Ge at 300 K	$0.661 \ {\rm eV}$
at 350 K	$0.641 \mathrm{~eV}$
at 380 K	$0.629 \ \mathrm{eV}$

Table 4: E_g found from resistivity behaviour in intrinsic conduction region. The error on E_g comes from the statistical error on the slope of the linear fit in figure 12. The results using V5 or V6 separately are shown for comparison. The expected values where computed using equation (23).

4.1.4 Extrinsic conduction region

Again, from figure 11, we expect the extrinsic conduction to be dominant until the resistivity stop increasing, i.e., before 300 K. In this region, the relevant relationship is expected to be a power law, as explained in the theory, section 2.3, equation (7). A log-log plot of the resistivity in function of temperature in the extrinsic region gives us the exponent, and is shown in figure 13. The resulting exponent is given in table 5. The small statistical error on the slope clearly indicates that the log-log relationship was linear. We find our exponent (1.715 \pm 0.005) to be closer to the simple approximation given in equation (7) (3/2) than the one measured for a Ge sample in Melissinos [5] (2.0 \pm 0.1). In any case, from this relationship, we can deduce the temperature dependance of the drift mobility, using (8):

$$\mu_d \propto T^{-1.715 \pm 0.005} \tag{24}$$

according to our measurements. We can compare this result with the expected exponent given in [8] for weakly doped germanium, which is -1.66 for the drift mobility of the electrons.

measured β_d (with ρ from 150 K to 285 K)	1.715 ± 0.005
using V5 only	1.717 ± 0.005
using V6 only	1.713 ± 0.005
expected exponent from the simple approx. in theory	1.5
expected exponent given by [8]	1.66
measured exponent in Melissinos $[5]$	2.0 ± 0.1

Table 5: β_d found from resistivity behaviour in extrinsic conduction region. The error on β_d comes from the statistical error on the slope of the linear fit in figure 13. The results using V5 or V6 separately are shown for comparison. The 'd' in β_d is for 'drift', and is used to distinguish it from β_H , computed using the Hall effect.



Figure 13: **Resistivity in extrinsic conduction region.** The comments for the error and the methodology for the plot are similar to the ones for figure 12. A linear fit was done through the blue points (region from 150 K to 285 K) and the red points were excluded (285 K to 320 K). The points in red boxes are explained in the text.

Using the temperature evolution given in figure 9 and the resistivity curve in figure 11, we can explain several characteristics of figure 13. First of all, the two points marked in a red box on figure 13 correspond to the temperature 209 K and 236 K, which are seen on figure 11 and figure 9 to correspond when the program had stopped (bugged), explaining why they are a bit off of the line on figure 13. Secondly, the first few points in figure 13 are also seen to be a bit more off of the line fit than the others. This can be explained by the fact that the temperature was increasing more rapidly in this region, yielding thus less stable result.

4.2 Hall Effect

4.2.1 Hall voltage

For the Hall voltage, we needed V1 and V2. As explained in section 4.1.1, we used the hall_process program to obtain the data in function of temperature. For the same reasons as in the last section, we had a data gap in the region of 165 K to 195 K, so we used a bin size of 3 K. We did two runs with a magnetic field of 0.449 ± 0.007 Tesla, one for each side. The raw data with bin size of 3 K is shown in figure 14. We see on this graph (especially for V2) that the voltage due to the magnetic field is not exactly reversed when the magnetic field changes sign. An offset voltage of the order of 10 mV can be computed using the average of the voltages on both side. This justifies very well why we needed to make this extra run.

The Hall voltage can be obtained by subtracting V1 (or V2) obtained with side 1 of V1 (or V2) obtained with a reversed field (side 2), as explained in section3.4:

$$V_H = \frac{V I_{\text{side } 1} - V 2_{\text{side } 2}}{2} \tag{25}$$

The Hall voltage obtained in function of temperature is shown in figure 15. We can notice on this graph a pronounced change in the behaviour around 310 K, which was said to separate the extrinsic conduction region to the intrinsic conduction region in our previous analysis of the resistivity. Also, we see that the V1 and V2 curves are within two standard deviations of each other, giving again a sanity check for our setup.

4.2.2 Hall mobility

Using the Hall voltage and the resistivity voltage (V5 or V6), we can compute the Hall mobility with equation 17 (a bit modified):

$$\mu_H = \frac{V_H}{V_R} \frac{l}{wB} \tag{26}$$

where l and w are the same as in figure 8, V_R is V5 or V6 (longitudinal voltage) and B is the strength of the exterior magnetic field. The ratio V_H/V_R is recognized to be the Hall angle, but is not used by itself. The V_H we used to

V1 and V2 with B = 0.493 T



Figure 14: V1 and V2 with B = 0.493 T. The random error was smaller than 1% and is not shown. The bin size used was 3 K.



Figure 15: Hall voltage in function of temperature.

Log-log plot of Hall mobility



Figure 16: Log-log plot of Hall mobility in function of temperature. The random errors were too small to be plotted here; the systematic error only affects the y-intercept, so is not included neither. A linear fit was done through the blue points (same region from 150 K to 285 K as in the resistivity case) and the red points were excluded.

compute the Hall mobility was the average of the ones obtained by V1 and V2. The V_R used was the average of V5 and V6 on both sides of the magnetic field (so 4 values). l and w were obtained by taking the average of L5, L6 and w1, w2 respectively. As in the resistivity section, we expect the mobility to have a power law dependance on temperature in the extrinsic conduction region. A log-log plot of the Hall mobility computed from equation (26) in function of temperature is plotted given in figure 16. The obtained exponent is given in table 6. It is called β_H since it was obtained with the Hall effect. We see that it is significantly different (by 0.333) than β_d (table 5), obtained from the resistivity measurements. This justifies well why we distinguish μ_d of μ_H . The difference between the two exponents is of the same order than the one obtained by other experimenters (for example, Melissinos had obtained 2.0 and 1.5 for β_d and β_H respectively). We can also compare our value of μ_H at 323 K with the one given in Melissinos [5, p.97]. Those are also shown in table 6. They are of the same order.

measured β_H (in extrinsic region from 150 K to 285 K)	1.382 ± 0.006
measured μ_H at 323 K	$0.20 \pm 0.02 \text{ Tesla}^{-1}$
μ_H at 323 K obtained in Melissinos	0.27 Tesla^{-1}

Table 6: **Results derived from Hall mobility.** The error on β_H comes from the statistical error on the slope of the linear fit in figure 16. The error on μ_H comes mainly from the systematic error arising on the measurements of the crystal dimension.

4.2.3 Hall coefficient

From equation (15) and (16), we had the following relationship for the Hall coefficient R_H :

$$R_H = \mu_H \rho = \frac{1}{ne} \tag{27}$$

where n was the density of negative charge carriers. We can witness the behaviour of R_H directly on figure 15, since by combining equation (13) and (26), we obtain

$$R_H = V_H \frac{A}{wIB} \propto V_H \tag{28}$$

since the other parameters are constant during the run. We see on this graph that R_H is approximately constant (increasing slightly) in the extrinsic conduction region. As explained in the theory, n was expected to be constant for the extrinsic region above 120 K, so R_H too should be constant there. But in our results, β_d is bigger than β_H , so R_H increases as $T^{\beta_d-\beta_H}$, implying that the simple models considered in the theory need some adjustments.

 R_H and n_{ext} computed at 300 K (for comparison with the specifications) are given in table 7. Both values are within a factor of two from the specifications, as usual.

measured R_H at 300 K	$(2.4 \pm 0.1) \times 10^4 \text{ cm}^3/\text{C}$
R_H given in specifications	$1.47 \times 10^4 \text{ cm}^3/\text{C}$
measured charge carrier concentration at 300 K	$(2.6 \pm 0.1) \times 10^{14} \text{ atoms/cm}^3$
concentration from specifications	$5 \times 10^{14} \text{ atoms/cm}^3$

Table 7: Results from Hall coefficient at 300 K.

Finally, since R_H doesn't go to 0 (see figure 15), we conclude that we have a *n*-type crystal. (see section 2.4.2)

4.3 Magnetoresistance

By measuring V5 for different magnetic fields, we can infer the resistance due to the magnetic field. According to equation (19), we expect

$$\rho(B) - \rho_0 \propto B^2 \tag{29}$$

Magnetoresistance



Figure 17: Variation of V5 in function of magnetic field. A power law was fitted using Excel (this gives the same result as a linear fit of the log-log relationship).

or

$$V5 - V5_0 \propto B^2 \tag{30}$$

since the resistivity is directly proportional to V5 (with fixed current in the sample). We thus subtracted the value of V5 when B = 0 (215.27 ± 0.01 mV) to the voltages we had measured. The differences obtained are plotted on log-log scale in figure 17. The exponent obtained by a linear fit of the log-log relationship is 1.78 ± 0.01 . This is "close"⁴ to the value of 2 expected by a quick approximation of the effect and given in [6]. The fact that we have taken the data very rapidly (to avoid effects of heating of the magnet) instead of taking it very very slowly (so to have a real equilibrium measurement) could be a source of error. We can also mention that the maximum magnetoresistance voltage measured indicates that roughly 5% of the resistivity was caused by the magnetic field. For a more precise analysis, this effect should be taken in consideration in the models used in the theory for this lab.

 $^{^{4}}$ In the scale of the exponents we have seen in this experiment...

5 Conclusion

This experiment showed how impurities, temperature and the presence of a magnetic field alter the mechanism of conduction. Measurements of the temperature dependence of its resistivity ρ together with the Hall effect revealed crucial properties of our sample. Our method allowed to discover the n-type nature of the dopant. Also, it provided an easy estimate of its concentration. Initially, the bandgap energy was not assumed to be temperature dependent. An undervalued E_g like ours was predicted from more advanced conduction theory. As in [5], two types of charge carriers' mobilities were observed. With a deeper understanding of collisions and interactions inside a material, the concepts of μ_d and μ_H should eventually make the correspondence between microscopic and measurable effects. Several power laws were found with a small statistical error on the exponent. The discrepancy with the theoretical models indicates that the phenomena under study are quite complex, and that Solid State Physics is first of all an experimental field.

Though this experiment represents a general qualitative success, it could be greatly improved on two fronts. First of all, the theoretical results could use less assumptions and take into account more parameters. This explains partially why for $\rho(T)$ and the quadratic dependence of magnetoresistance, we were only able to show a general behavior and infirm no quantitative values. On the other hand, some modifications would reduce evident sources of errors; for example, data taken over shorter cycles, a clean cut sample, longer runs with quasi thermal equilibrium, greater range of temperature allowed by the soldering, etc.

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